UNIT-II

INTERPOLATION & APPROXIMATION

LAGRANGE POLYNAMIAL

1. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x :	0	1	2	5
$f(x)$:	2	3	12	147

Solution :

x :	0 x_0	1 x_1	2 x_2	5 x_3
$f(x)$:	0 y_0	3 y_1	12 y_3	147 y_4

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\, y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\, y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\, y_3$$

$$f(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$f(x) = \frac{(x-1)(x-2)(x-5)}{-10}(2) + \frac{(x-0)(x-2)(x-5)}{4}(3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{-6}(12) + \frac{(x-0)(x-1)(x-2)}{60}(147)$$

To find $f(3)$: $(x = 3)$:

$$y = f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{(3-0)(3-2)(3-5)}{4}(3)$$

$$+ \frac{(3-0)(3-1)(3-5)}{-6}(12) + \frac{(3-0)(3-1)(3-2)}{60}(147)$$

$$y = f(3) = 0.8 - 4.5 + 24 + 14.7$$

$$\boldsymbol{y = f(3) = 35.}$$

2. Using Lagrange's interpolation formula, calculate the profit in the 2000 year from the following data

Year	1997	1999	2001	2002
Profit in Lakhs:	43	65	159	248

Solution : Given the data's are

Year	1997 (x_0)	1999 (x_1)	2001 (x_2)	2002 (x_3)
Profit in Lakhs:	43 (y_0)	65 (y_1)	159 (y_2)	248 (y_3)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\, y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\, y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\, y_3$$

$$f(x) = \frac{(x-1999)(x-2001)(x-2002)}{(1997-1999)(1997-2001)(1997-2002)}(43) + \frac{(x-1997)(x-2001)(x-2002)}{(1999-1997)(1999-2001)(1999-2002)}(65)$$

$$+ \frac{(x-1997)(x-1999)(x-2002)}{(2001-1997)(2001-1999)(2001-2002)}(159) + \frac{(x-1997)(x-1999)(x-2001)}{(2002-1997)(2002-1999)(2002-2001)}(43)$$

$$f(x) = \frac{(x-1999)(x-2001)(x-2002)}{-40}(43) + \frac{(x-1997)(x-2001)(x-2002)}{12}(65)$$

$$+ \frac{(x-1997)(x-1999)(x-2002)}{-8}(159) + \frac{(x-1997)(x-1999)(x-2001)}{15}(43)$$

To find the profit in the year 2000 ($\boldsymbol{f(2000)}$: $(\boldsymbol{x = 2000})$:)

$$y = f(2000) = \frac{(2000-1999)(2000-2001)(2000-2002)}{-40}(43)$$

$$+ \frac{(2000-1997)(2000-2001)(2000-2002)}{12}(65)$$

$$+ \frac{(2000-1997)(2000-1999)(2000-2002)}{-8}(159)$$

$$+ \frac{(2000-1997)(2000-1999)(2000-2001)}{15}(43)$$

$$y = f(2000) = -2.15 + 32.5 + 119.25 - 49.6$$

$$y = f(2000) = 100$$

Hence the profit in the year **2000** is **100**.

3. Using Lagrange's formula find $y(2)$ from the following data.

x :	0	1	3	4	5
$f(x)$:	0	1	81	256	625

Solution :

x :	0 (x_0)	1 (x_1)	3 (x_2)	4 (x_3)	5 (x_4)
$f(x)$:	0 (y_0)	1 (y_1)	81 (y_2)	256 (y_3)	625 (y_4)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\, y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\ y_2+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\ y_3$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}\ y_4$$

$$f(x)=\frac{(x-1)(x-3)(x-4)(x-5)}{(0-1)(0-3)(0-4)(0-5)}\ (0)+\frac{(x-0)(x-3)(x-4)(x-5)}{(1-0)(1-3)(1-x_3)(1-5)}\ (1)$$

$$+\frac{(x-0)(x-1)(x-4)(x-5)}{(3-0)(3-1)(3-4)(3-5)}\ (81)+\frac{(x-0)(x-1)(x-3)(x-5)}{(4-0)(4-1)(4-3)(4-5)}\ (256)$$

$$+\frac{(x-0)(x-1)(x-3)(x-4)}{(4-0)(4-1)(4-3)(5-4)}\ (625)$$

To fond $y(2):(x=2):$

$$y=f(2)=\frac{(2-1)(2-3)(2-4)(2-5)}{-48}\ (0)+\frac{(2-0)(2-3)(2-4)(2-5)}{-24}\ (1)$$

$$+\frac{(2-0)(2-1)(2-4)(2-5)}{12}\ (81)+\frac{(2-0)(2-1)(2-3)(2-5)}{-12}\ (256)$$

$$+\frac{(2-0)(2-1)(2-3)(2-4)}{40}\ (625)$$

$$y=f(2)=0+0.5+81-128+62.5$$

$$y=f(2)=16.$$

4. **Find the third degree polynomial satisfying the following data**

x :	1	3	5	7
$f(x)$:	24	120	336	720

Solution :

x :	1 (x_0)	3 (x_1)	5 (x_2)	7 (x_3)
$f(x)$:	24 (y_0)	120 (y_1)	336 (y_2)	720 (y_3)

Lagrange's interpolation formula, we have

$$y=f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\ y_0+\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\ y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\ y_2+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\ y_3$$

$$f(x)=\frac{(x-3)(x-5)(x-7)}{(1-3)(1-5)(1-7)}\ (24)+\frac{(x-1)(x-5)(x-7)}{(3-1)(3-5)(3-7)}\ (120)+\frac{(x-1)(x-3)(x-7)}{(5-1)(5-3)(5-7)}\ (336)$$

$$+\frac{(x-1)(x-3)(x-5)}{(7-1)(7-3)(7-5)}\ (720)$$

$$f(x) = \frac{-1}{2}\,[(x-3)(x-5)(x-7)] + \frac{15}{2}[(x-1)(x-5)(x-7)] - 21\,[(x-1)(x-3)(x-7)] + 15\,[(x-1)(x-3)(x-5)]$$

$$f(x) = \frac{-1}{2}\,[x^3 - 15\,x^2 + 71\,x - 105] + \frac{15}{2}[x^3 - 13\,x^2 + 47\,x - 35] - 21\,[x^3 - 11\,x^2 + 31\,x - 21] + 15\,[x^3 - 9\,x^2 + 23\,x - 15]$$

$$f(x) = x^3\left[-\frac{1}{2} + \frac{15}{2} - 21 + 15\right] + x^2\left[\frac{15}{2} - \frac{195}{2} + 231 - 135\right] + x\left[-\frac{71}{2} + \frac{705}{2} - 605 + 345\right] + \left[\frac{105}{2} - \frac{525}{2} + 441 - 225\right]$$

$f(x) = x^3 + 6\,x^2 + 11\,x + 6.$

$f(4) = (4)^3 + 6\,(4)^2 + 11\,(4) + 6.$

$f(4) = 64 + 96 + 44 + 6$

$f(4) = 210.$

5. Using Lagrange's interpolation formula find $f(4)$ given that $f(0) = 2,\ f(1) = 3,\ f(2) = 12,\ f(15) = 3587.$

Solution : Given the data's are $[f(x_0) = y_0]$

x :	0 (x_0)	1 (x_1)	2 (x_2)	15 (x_3)
$f(x)$:	2 (y_0)	3 (y_1)	12 (y_2)	3587 (y_3)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\,y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\,y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\,y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\,y_3$$

$$y = f(4) = \frac{(4-1)(4-2)(4-15)}{(0-1)(0-2)(0-15)}\,(2) + \frac{(4-0)(4-2)(4-15)}{(1-0)(1-2)(1-15)}\,(3) + \frac{(4-0)(4-1)(4-15)}{(2-0)(2-1)(2-15)}\,(12) + \frac{(4-0)(4-1)(4-2)}{(15-0)(15-1)(15-2)}\,(3587)$$

$$f(4) = \frac{(3)(2)(-11)}{(-1)(-2)(-15)}\,(2) + \frac{(4)(2)(-11)}{(1)(-1)(-14)}\,(3) + \frac{(4)(3)(-11)}{(2)(1)(-13)}(12) + \frac{(4)(3)(2)}{(15)(14)(13)}(3587)$$

$$f(4) = \frac{132}{30} - \frac{264}{14} + \frac{1584}{26} + \frac{86088}{2730}$$

$\boldsymbol{f(4) = 78}$

6. Using Lagrange's polynomial fit a polynomial for the following data

x :	−1	1	2
$f(x)$:	7	5	15

Solution :

x :	-1 (x_0)	1 (x_1)	2 (x_2)
$f(x)$:	7 (y_0)	5 (y_1)	15 (y_2)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\, y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\, y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\, y_2$$

$$f(x) = \frac{(x-1)(x-2)}{(-1-1)(-1-2)}\,(7) + \frac{(x+1)(x-2)}{(1+1)(1-2)}\,(5) + \frac{(x+1)(x-1)}{(2+1)(2-1)}\,(15)$$

$$f(x) = \frac{7}{6}\,[\,x^2 - 3\,x + 2] - \frac{5}{2}[\,x^2 - \,x - 2] + 5\,[\,x^2 - 1]$$

$$f(x) = x^2\left[\frac{7}{6} - \frac{5}{2} + 5\,\right] + x\left[-\frac{7}{2} + \frac{5}{2}\right] + \left[\frac{7}{3} + 5 - 5\right]$$

$$f(x) = \left(\frac{22}{6}\right) x^2 - \,x + \left(\frac{7}{3}\right).$$

$$\boldsymbol{f(4) = \frac{1}{3}\,[11\,x^2 - 3\,x + 7].}$$

7. Find the missing term in the following table using Lagrange's interpolation.

$\boldsymbol{x}$:	0	1	2	3	4
$\boldsymbol{f(x)}$:	1	3	9	--	81

Solution :

x :	0 (x_0)	1 (x_1)	2 (x_2)	4 (x_3)
$f(x)$:	1 (y_0)	3 (y_1)	9 (y_2)	81 (y_3)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\, y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\, y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\, y_3$$

$$y = f(x) = \frac{(x-1)(x-2)(x-4)}{(0-1)(0-2)(0-4)}\,(1) + \frac{(x-0)(x-2)(x-4)}{(1-0)(1-2)(1-4)}\,(3)$$

$$+ \frac{(x-0)(x-1)(x-4)}{(2-0)(2-1)(2-4)}\,(9) + \frac{(x-0)(x-1)(x-2)}{(4-0)(4-1)(4-2)}\,(81)$$

$$f(3) = \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)}\,(1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)}\,(3)$$

$$+ \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)}\,(9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)}\,(81)$$

$$f(3) = \frac{(2)(1)(-1)}{(-1)(-2)(-4)}(1) + \frac{(3)(1)(-1)}{(1)(-1)(-3)}(3) + \frac{(3)(2)(-1)}{(2)(1)(-2)}(9) + \frac{(3)(2)(1)}{(4)(3)(2)}(81)$$

$$f(3) = -\frac{2}{8} - 3 + \frac{27}{2} + \frac{81}{4}$$

$$\boldsymbol{f(3) = 31.}$$

DIVIDED DIFFERNCES

1. **Using Newton's divided difference formula, find $u(3)$ given $u(1) = -26, u(2) = 12,\ u(4) = 256,\ u(6) = 844$.**

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	-26			
		$\frac{12+26}{2-1} = 38$		
			$\frac{122-38}{4-1} = -3.6$	
2	12			
		$\frac{256-12}{4-2} = 122$		$\frac{43-28}{6-1} = 3$
			$\frac{294-122}{6-2} = 43$	
4	256			
		$\frac{844-256}{6-4} = 294$		
6	844			

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)f(x_0,x_1) + (x - x_0)(x - x_1)f(x_0,x_1,x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0,x_1,x_2,x_3)$$

Here $x_0 = 1,\ x_1 = 2,\ x_2 = 4,\ x_3 = 6$

And $f(x_0) = -26,\ f(x_0,x_1) = 38,\ f(x_0,x_1,x_2) = 28,\ f(x_0,x_1,x_2,x_3) = 3$

$$\therefore\ f(x) = -26 + (x-1)\ 38 + (x-1)(x-2)\ 28 + (x-1)(x-2)(x-4)\ 3$$

$$\therefore\ f(3) = -26 + (3-1)\ 38 + (3-1)(3-2)\ 28 + (3-1)(3-2)(3-4)\ 3$$

$$f(3) = -26 + (2)\ 38 + (2)(1)\ 28 + (2)(1)(-1)\ 3$$

$$f(3) = 100$$

2. **Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula**

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$

-4	1245	$\frac{33-1245}{-1-(-4)} = -404$			
			$\frac{-28-(-404)}{0-(-4)} = 94$		
-1	33				
		$\frac{5-33}{0-(-1)} = -28$		$\frac{10-94}{2-(-4)} = -14$	
			$\frac{2-(-28)}{2-(-1)} = 10$		$\frac{13+14}{5-(-4)} = 3$
0	5				
		$\frac{9-5}{2-0} = 2$		$\frac{88-10}{5-0} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
5	1335	$\frac{1335-9}{5-2} = 442$			

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)f(x_0,x_1,x_2,x_3,x_4)$$

Here $x_0 = -4,\ x_1 = -1,\ x_2 = 0,\ x_3 = 2,\ x_4 = 5,\ and$

$$\boldsymbol{f(x_0) = 1245,\ f(x_0,x_1) = -404,\ f(x_0,x_1,x_2) = 94,\ f(x_0,x_1,x_2,x_3) = -14,\ f(x_0,x_1,x_2,x_3,x_4) = 3}$$

$$\therefore\quad f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x-0)(-14) + (x+4)(x+1)(x-0)(x-2)(3)$$

$$f(x) = 1245 - 404x - 1616 + (94)[x^2+5x+4] - 14x[x^2+5x+4] + 3x[(x^2+5x+4)(x-2)]$$

$$f(x) = 1245 - 404x - 1616 + 94x^2 + 470x + 376 - 14x^3 - 70x^2 - 56x + 3x[x^3 - 2x + 5x^2 - 10x + 4x - 8]$$

$$f(x) = -14x^3 + 24x^2 + 10x + 5 + 3x[x^3 + 5x^2 - 8x - 8]$$

$$f(x) = -14x^3 + 24x^2 + 10x + 5 + 3x^4 + 15x^3 - 24x^2 - 24x$$

$$f(x) = 3x^4 + x^3 - 14x + 5$$

3. **Find $f(8)$ by Newton's divided difference formula for the data,**

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$	
					0
5	100	$\frac{294-100}{5-7} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$	
					0
7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$	

10	900	$\frac{1210-900}{11-10}=310$	$\frac{409-310}{13-10}=33$		
11	1210	$\frac{2028-1210}{13-11}=409$			
13	2028				

By Newton's divided difference interpolation formula

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$
$$+(x-x_0)(x-x_1)(x-x_2)(x-x_3)f(x_0,x_1,x_2,x_3,x_4)$$

Here $x_0=4,\ x_1=5,\ x_2=7,\ x_3=10,\ x_4=11, x_5=13\ and$

$$f(x_0)=48,\ \ f(x_0,x_1)=52,\ \ f(x_0,x_1,x_2)=15,\ \ f(x_0,x_1,x_2,x_3)=1,\ f(x_0,x_1,x_2,x_3,x_4)=0$$

$$\therefore\quad f(x)=48+(x-4)\,(52)+(x-4)(x-5)\,(15)+(x-4)(x-5)(x-7)\,(1)$$
$$+(x-4)(x-5)(x-7)(x-11)(0)+0$$

$$\therefore\quad f(8)=48+(8-4)\,(52)+(8-4)(8-5)\,(15)+(8-4)(8-5)(8-7)\,(1)+0$$

$$f(8)=48+(4)\,(52)+(4)(3)\,(15)+(4)(3)(1)\,(1)$$

$$f(8)=448.$$

4. **Find $f(3)$ by Newton's divided difference formula for the data,**

x :	0	1	2	4	5
$f(x)$:	1	14	15	5	6

Solution :

x	$f(x)$	$\Delta\, f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1				
		$\frac{14-1}{1-0}=13$			
1	14		$\frac{1-13}{2-0}=-6$		
		$\frac{15-14}{2-1}=1$		$\frac{-2+6}{4-0}=1$	
2	15		$\frac{-5-1}{4-1}=-2$		$\frac{1-1}{5-0}=0$
		$\frac{5-15}{4-2}=-5$		$\frac{2+2}{5-1}=1$	
4	5		$\frac{1+5}{5-2}=2$		
		$\frac{6-5}{5-4}=1$			
5	6				

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)f(x_0,x_1) + (x - x_0)(x - x_1)f(x_0,x_1,x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0,x_1,x_2,x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)f(x_0,x_1,x_2,x_3,x_4)$$

Here $x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 4,\ x_4 = 5\ \ and$

$f(x_0) = 1,\ f(x_0,x_1) = 13,\ f(x_0,x_1,x_2) = -6,\ \ f(x_0,x_1,x_2,x_3) = 1,\ f(x_0,x_1,x_2,x_3,x_4) = 0$

$$\therefore\ \ f(x) = 1 + (x - 0)\,(13) + (x - 0)(x - 1)\,(-6) + (x - 0)(x - 1)(x - 2)\,(1) + (x - 0)(x - 1)(x - 2)(x - 4)(0) + 0$$

$$\therefore\ \ f(3) = 1 + (3 - 0)\,(13) + (3 - 0)(3 - 1)\,(-6) + (3 - 0)(3 - 1)(3 - 2)\,(1) + 0$$

$$f(3) = 1 + (3)\,(13) + (3)(2)\,(-6) + (3)(2)(1)\,(1)$$

$$f(3) = 10$$

5. **Using Newton's divided difference formula, find the missing term in the following data**

$x:$	1	2	4	5	6
$f(x)$:	14	15	5	--	9

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	14			
		$\frac{15-14}{2-1} = 1$		
2	15		$\frac{-5-1}{4-1} = -2$	
		$\frac{5-15}{4-2} = -5$		$\frac{1.75+2}{6-1} = 0.75$
4	5		$\frac{2+5}{6-2} = 1.75$	
		$\frac{9-5}{6-4} = 2$		
6	9			

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)f(x_0,x_1) + (x - x_0)(x - x_1)f(x_0,x_1,x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0,x_1,x_2,x_3)$$

Here $x_0 = 1,\ x_1 = 2,\ x_2 = 4,\ x_3 = 6$

And $f(x_0) = 14,\ f(x_0,x_1) = 1,\ f(x_0,x_1,x_2) = -2,\ \ f(x_0,x_1,x_2,x_3) = 0.75$

$$\therefore\ \ f(x) = 14 + (x - 1)\,(1) + (x - 1)(x - 2)\,(-2) + (x - 1)(x - 2)(x - 4)\ \ (0.75)$$

$$\therefore\ \ f(5) = 14 + (5 - 1)\,(1) + (5 - 1)(5 - 2)\,(-2) + (5 - 1)(5 - 2)(5 - 4)\,(0.75)$$

$$f(5) = 14 + (4)\,(1) + (4)(3)\,(-2) + (4)(3)(1)\,(0.75)$$

$$f(5) = 3.$$

NEWTONS FORWARD & BACKWARD INTERPOLATION FORMULA

Newton forward interpolation formula :

$$y\,(x) = y\,(x_0 + ph) = y_0 + \frac{u}{1\,!}\,\Delta\, y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\, y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\, y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\, y_0 + \cdots \ldots \ldots \qquad where \quad u = \frac{x - x_0}{h}$$

Newton backward interpolation formula :

$$y\,(x) = y\,(x_n + ph) = y_n + \frac{V}{1\,!}\,\nabla\, y_n + \frac{V(V+1)}{2\,!}\,\nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\,\nabla^3 y_n + \frac{V(V+1)(V+2)(V+3)}{4!}\,\nabla^4 y_n + \cdots \ldots \ldots \qquad where \quad u = \frac{x - x_n}{h}$$

1. **Using Newton's forward interpolation formula, find a polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x = 5$.**

x :	4	6	8	10
y :	1	3	8	10

Solution : We form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$4\ (x_0)$	$1\ \ (y_0)$			
		$3 - 1 = \mathbf{2}\ \ (\Delta y_0)$		
$6\ (x_1)$	$3\ \ (y_1)$		$5 - 2 = \mathbf{3}\ \ (\Delta^2 y_0)$	
		$8 - 3 = 5\ \ (\Delta y_1)$		$\mathbf{-6}\ \ (\Delta^3 y_0)$
$8\ (x_2)$	$8\ \ (y_2)$		$2 - 5 = -3\ \ (\Delta^2 y_1)$	
		$10 - 8 = 2\ \ (\Delta y_2)$		
$10\ (x_3)$	$10\ \ (y_3)$			

There are only four data are given. Hence the polynomial is of order 3

$$y\,(x) = y_0 + \frac{u}{1\,!}\,\Delta\, y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\, y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\, y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\, y_0 + \cdots$$

$where \quad u = \dfrac{x - x_0}{h}$. $Here\ x_0 = 4 \ \ \&\ \ h = 6 - 4 = 2 \quad (or) \quad 10 - 8 = 2$

$Let \quad u = \left(\frac{x-4}{2}\right)$

$$y\,(x) = 1 + \frac{\left(\frac{x-4}{2}\right)}{1\,!}(2) + \frac{\left(\frac{x-4}{2}\right)\left[\left(\frac{x-4}{2}\right) - 1\right]}{2\,!}(3) + \frac{\left(\frac{x-4}{2}\right)\left[\left(\frac{x-4}{2}\right) - 1\right]\left[\left(\frac{x-4}{2}\right) - 2\right]}{3\,!}(-6)$$

$$= 1 + \frac{\left(\frac{x-4}{2}\right)}{1}(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{2}(3) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)\left(\frac{x-8}{2}\right)}{6}(-6)$$

$$= 1 + \frac{x-4}{2}(2) + \left(\frac{3}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right) + \left(-\frac{6}{6}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)\left(\frac{x-8}{2}\right)$$

$$= 1 + x - 4 + \left(\frac{3}{2}\right)\left(\frac{1}{4}\right)(x-4)(x-6) + (-1)\left(\frac{1}{8}\right)(x-4)(x-6)(x-8)$$

$$= x - 3 + \left(\frac{3}{8}\right)(x^2 - 4x - 6x + 24) + \left(\frac{-1}{8}\right)(x^2 - 4x - 6x + 24)(x-8)$$

$$= x - 3 + \left(\frac{3}{8}\right)(x^2 - 10x + 24) + \left(\frac{-1}{8}\right)(x^2 - 10x + 24)(x-8)$$

$$= x - 3 + \left(\frac{3}{8}\right)(x^2 - 10x + 24) + \left(\frac{-1}{8}\right)(x^3 - 10x^2 + 24x - 8x^2 + 80x - 192)$$

$$= x - 3 + \left(\frac{3}{8}\right)(x^2 - 10x + 24) + \left(\frac{-1}{8}\right)(x^3 - 18x^2 + 104x - 192)$$

$$= x - 3 + \left(\frac{1}{8}\right)[3(x^2 - 10x + 24) - (x^3 - 18x^2 + 104x - 192)]$$

$$= x - 3 + \left(\frac{1}{8}\right)[-x^3 + 21x^2 - 134x + 264]$$

$$y(x) = \left(\frac{1}{8}\right)(-x^3 + 21x^2 - 126x + 240)$$

To find y at $x = 5$:

$$y(5) = \left(\frac{1}{8}\right)[-(5)^3 + 21(5)^2 - 126(5) + 240]$$

$$y(5) = \left(\frac{1}{8}\right)(10) = 1.25$$

Verification : $f(6) = \left(\frac{1}{8}\right)[-(6)^3 + 21(6)^2 - 126(6) + 240] \Rightarrow \left(\frac{1}{8}\right)[-216 + 756 - 756 + 240]$

$$f(5) = \left(\frac{1}{2}\right)[24] \Rightarrow 3$$

Therefore the polynomial is correct.

2. **Using Newton's forward formula, find a polynomial $f(x)$ satisfying the following data. Hence find $f(2)$.**

x :	0	5	10	15
y :	14	379	1444	3584

Solution : We form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$0\ (x_0)$	$14\ (y_0)$			
		$379 - 14 = \mathbf{365}\ (\Delta y_0)$		
$5\ (x_1)$	$379\ (y_1)$		$\mathbf{700}\ (\Delta^2 y_0)$	
		$1065\ (\Delta y_1)$		$\mathbf{375}\ (\Delta^3 y_0)$
$10\ (x_2)$	$1444\ (y_2)$		$1075\ (\Delta^2 y_1)$	
		$2140\ (\Delta y_2)$		
$15\ (x_3)$	$3584\ (y_3)$			

There are only four data are given. Hence the polynomial is of order 3

$$y\,(x) = y\,(x_0 + ph) = y_0 + \frac{u}{1\,!}\,\Delta\, y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\, y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\, y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\, y_0 + \cdots \ldots\ldots$$

$$where\ \ u = \frac{x - x_0}{h}\ \ .\ Here\ \ x_0 = 0\ \ \&\ \ h = 5 - 0 = 5\ \ \ (or)\ \ \ 10 - 5 = 5$$

$$Let\ \ \ u = \left(\frac{x-0}{5}\right) = \frac{x}{5}$$

$$y\,(x) = 14 + \frac{\left(\frac{x}{5}\right)}{1\,!}\,(365) + \frac{\left(\frac{x}{5}\right)\left[\left(\frac{x}{5}\right) - 1\right]}{2\,!}\,(700) + \frac{\left(\frac{x}{5}\right)\left[\left(\frac{x}{5}\right) - 1\right]\left[\left(\frac{x}{5}\right) - 2\right]}{3\,!}\,(375)$$

$$= 14 + \left(\frac{x}{5}\right)\,(365) + \frac{\left(\frac{x}{5}\right)\left(\frac{x-5}{5}\right)}{2}\,(700) + \frac{\left(\frac{x}{5}\right)\left(\frac{x-5}{5}\right)\left(\frac{x-10}{5}\right)}{6}\,(375)$$

$$= 14 + (x)\,(73) + \left(\frac{x}{5}\right)\left(\frac{x-5}{5}\right)(350) + \left(\frac{x}{5}\right)\left(\frac{x-5}{5}\right)\left(\frac{x-10}{5}\right)\left(\frac{1}{6}\right)(375)$$

$$= 14 + 73\,x + \left(\frac{350}{25}\right)[x(x-5)]\ + \left(\frac{1}{6}\right)\left(\frac{375}{125}\right)x(x-5)(x-10)$$

$$= 14 + 73\,x + 14\,(x^2 - 5x)\ + \left(\frac{1}{6}\right)3\,[(x^2 - 5x)(x - 10)]$$

$$= 14 + 73\,x + 14\,x^2 - 70\,x\ + \left(\frac{1}{2}\right)\,[(x^3 - 5x^2 - 10x^2 + 50\,x)]$$

$$= \left(\frac{1}{2}\right)\left[2(14 + 73\,x + 14\,x^2 - 70\,x\,) + \ [(x^3 - 5x^2 - 10x^2 + 50\,x)]\right]$$

$$y(x) = \left(\frac{1}{2}\right)[x^3 + 13x^2 + 56\,x + 28]$$

To find $f(2)$ **:**

$$f(2) = \left(\frac{1}{2}\right)[(2)^3 + 13(2)^2 + 56\,(2) + 28]$$

$$f(5) = \left(\frac{1}{2}\right)(200\,) \quad = 100$$

Verification : $f(5) = \left(\frac{1}{2}\right)[(5)^3 + 13(5)^2 + 56\,(5) + 28] \;\Rightarrow\; \left(\frac{1}{2}\right)[125 + 325 + 280 + 28]$

$$f(5) = \left(\frac{1}{2}\right)[758] \;\Rightarrow\; 379$$

3. **A Third degree polynomial passes through the points $(0,-1),(1,1),(2,1,)$ & $(3,-2)$ using Newton's forward interpolation formula find the polynomial. Hence evaluate the value at 1.5**

Solution : Let us form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$0\ (x_0)$	$-1\ (y_0)$			
		$1-(-1) = \mathbf{2}\ (\Delta y_0)$		
$1\ (x_1)$	$1\ (y_1)$		$0-2 = \mathbf{-2}\ (\Delta^2 y_0)$	
		$1-1 = 0\ (\Delta y_1)$		$-3-(-2) = \mathbf{-1}\ (\Delta^3 y_0)$
$2\ (x_2)$	$1\ (y_2)$		$-3-0 = -3\ (\Delta^2 y_1)$	
		$-2-1 = -3\ (\Delta y_2)$		
$3\ (x_3)$	$-2\ (y_3)$			

There are only four data are given. Hence the polynomial is of order 3

$$y\,(x) = y_0 + \frac{u}{1\,!}\,\Delta\, y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\, y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\, y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\, y_0 + \cdots \ldots \ldots$$

$$where \quad u = \frac{x - x_0}{h} \quad . \quad Here \quad x_0 = 0 \;\; \& \;\; h = 1 - 0 = 1 \quad (or) \quad 2 - 1 = 1$$

$$Let \quad u = \left(\frac{x-0}{1}\right) = x \;\Rightarrow\; u = x$$

$$y\,(x) = -1 + \frac{(x)}{1\,!}\,(2) + \frac{(x)[x-1]}{2\,!}\,(-2) + \frac{(x)[x-1][x-2]}{3\,!}\,(-1)$$

$$= -1 + x\,(2) - (x)[x-1] + \left(\frac{-1}{6}\right)(x)[x-1][x-2]$$

$$= -1 + 2\,x - [x^2 - x] + \left(\frac{-1}{6}\right)[x^2 - x][x-2]$$

$$= -x^2 + x - 1 + 2\,x + \left(\frac{-1}{6}\right)[x^3 - x^2 - 2x^2 + 2x]$$

$$= -x^2 + 3x - 1 + \left(\frac{-1}{6}\right)[x^3 - 3x^2 + 2x]$$

$$= \left(\frac{1}{6}\right)\left[6(-x^2 + 3x - 1) + (-1)[x^3 - 3x^2 + 2x]\right]$$

$$= \left(\frac{1}{6}\right)[-6x^2 + 18x - 6 - x^3 + 3x^2 - 2x]$$

$$y(x) = \left(\frac{1}{6}\right)[-x^3 - 3x^2 + 16x - 6]$$

To find y at 1.5 : $f(1.5) = \left(\frac{1}{6}\right)[-(1.5)^3 - 3(1.5)^2 + 16(1.5) - 6]$

$$f(1.5) = \left(\frac{1}{6}\right)[-3.375 - 6.75 + 24 - 6]$$

$$f(1.5) = \left(\frac{1}{6}\right)(7.875) = 1.3125$$

Verification : $f(2) = \left(\frac{1}{6}\right)[-(2)^3 - 3(2)^2 + 16(2) - 6] \Rightarrow \left(\frac{1}{6}\right)[-8 - 12 + 32 - 6]$

$$f(2) = \left(\frac{1}{6}\right)[+6] = 1$$

4. **Using Newton's forward interpolation formula find the polynomial which takes places the values**

x :	0	1	2	3
y :	1	2	1	10

Evaluate $f(4)$ using Newton's backward interpolation formula. Is it the same as obtained from the cubic polynomial found above.

Solution : Let us form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$0\ (x_0)$	$1\ (y_0)$			
		$[\nabla y_1]\ \ 1\ (\Delta y_0)$		
$1\ (x_1)$	$2\ (y_1)$		$[\nabla^2 y_2]\ \ -2\ (\Delta^2 y_0)$	
		$[\nabla y_2]\ \ -1\ (\Delta y_1)$		$[\nabla^3 y_3]\ \ \mathbf{12}\ (\Delta^3 y_0)$
$2\ (x_2)$	$1\ (y_2)$		$[\nabla^2 y_3]\ \ \mathbf{10}\ (\Delta^2 y_1)$	
		$[\nabla y_3]\ \ \mathbf{9}\ (\Delta y_2)$		
$3\ (x_3)$	$10\ (y_3)$			

There are only four data are given. Hence the polynomial is of order 3

The Newton's forward interpolation formula is

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$where \quad u = \dfrac{x - x_0}{h}$. $Here \quad x_0 = 0 \quad \& \quad h = 1 - 0 = 1 \quad (or) \quad 2 - 1 = 1$

$Let \quad u = \left(\frac{x-0}{1}\right) = x \implies u = x$

$$y\,(x) = +1 + \frac{(x)}{1\,!}\,(1) + \frac{(x)[x-1]}{2\,!}\,(-2) + \frac{(x)[x-1][x-2]}{3\,!}\,(12)$$

$$= 1 + x - (x)[x-1] + \left(\frac{12}{6}\right)(x)[x-1][x-2]$$

$$= 1 + x - [x^2 - x] + (2)[x^2 - x][x-2]$$

$$= -x^2 + x + 1 + x + (2)[x^3 - x^2 - 2x^2 + 2x]$$

$$= -x^2 + 2x + 1 + (2)[x^3 - 3x^2 + 2x]$$

$$= -x^2 + 2x + 1 + 2x^3 - 6x^2 + 4x$$

$$= 2x^3 - 7x^2 + 6x + 1$$

$$y(x) = 2x^3 - 7x^2 + 6x + 1$$

To find $y(4)$:

$$f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1 \quad = 2(64) - 7(16) + 6(4) + 1$$

$$f(4) = 41$$

Verification : $f(2) = 2(2)^3 - 7(2)^2 + 6(2) + 1 \implies = 16 - 28 + 12 + 1$

$$f(2) = +1$$

The Newton's backward interpolation formula is

$$y\,(x) = y\,(x_n + ph)\ = y_n + \frac{V}{1\,!}\,\nabla\, y_n + \frac{V(V+1)}{2\,!}\,\nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\,\nabla^3 y_n + \cdots \ldots\ldots$$

$where\ \ u = \frac{x - x_n}{h}$. Here $x_3 = 3$ & $h = 1 - 0 = 1\ \ (or)\ \ 2 - 1 = 1$

$Let\ \ u = \left(\frac{x-3}{1}\right) = x - 3 \ \Rightarrow\ u = (x-3)$

$$y\,(x) = 10 + \frac{(x-3)}{1\,!}\,(9) + \frac{(x-3)[(x-3)+1]}{2\,!}\,(10) + \frac{(x-3)[(x-3)+1][(x-3)+2]}{3\,!}\,(12)$$

$$= 10 + 9\,(x-3) + \left(\frac{10}{2}\right)(x-3)(x-2) + \left(\frac{12}{6}\right)(x-3)(x-2)(x-1)$$

$$= 9x - 17 + \ (5)(x^2 - 3x - 2x + 6) + \ (2)(x^2 - 3x - 2x + 6)(x-1)$$

$$= 9x - 17 + \ (5x^2 - 15x - 10x + 30) + \ (2)(x^3 - 3x^2 - 2x^2 + 6x - x^2 + 3x + 2x - 6)$$

$$= 9x - 17 + \ 5x^2 - 25x + 30 + \ (2)(x^3 - 6x^2 + 11x - 6)$$

$$= 9x - 17 + \ 5x^2 - 25x + 30 + \ 2x^3 - 12x^2 + 22x - 12$$

$$y(x) = 2x^3 - 7x^2 + 6x + 1$$

To find $y(4)$:

$$f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$$

$$f(4) = 41$$

Therefore the cubic polynomial in both cases are correct.

5. **Using Newton's backward formula find the interpolating polynomial of degree 3 for the data.**
   $\boldsymbol{f(-0.75) = -0.07181250,\ f(-0.5) = -0.024750,\ f(-0.25) = 0.33493750,\ f(0) = 1.10100.}$
   Hence find $\boldsymbol{f\left(\frac{-1}{3}\right)}$**.**

Solution : Since $f(x) = y$. Let us form the difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$

$-0.75\ (x_0)$	$-0.07181250\ \ (y_0)$			
		$[\nabla y_1]\ \ 0.0470625$		
$-0.5\ (x_1)$	$-0.024750\ \ (y_1)$		$[\nabla^2 y_2]\ \ 0.312625$	
		$[\nabla y_2]\ \ 0.3596875$		$[\nabla^3 y_3]\ \ \mathbf{0.09375}$
$-0.25\ (x_2)$	$0.33493750\ \ (y_2)$		$[\nabla^2 y_3]\ \ \mathbf{0.400375}$	
		$[\nabla y_3]\ \ \mathbf{0.7660625}$		
$0\ \ (x_3)$	$1.10100\ \ (y_3)$			

There are only four data are given. Hence the polynomial is of order 3

The Newton's backward interpolation formula is

$$y\,(x) = y\,(x_n + ph) \ = y_n + \frac{V}{1\,!}\,\nabla\,y_n + \frac{V(V+1)}{2\,!}\,\nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\,\nabla^3 y_n + \cdots \ldots\ldots$$

$where\ \ u = \frac{x - x_n}{h}$. Here $x_3 = 0$ & $h = -0.50 - (-0.75) = +0.25$

$Let\ \ \ u = \left(\frac{x-0}{0.25}\right) = \frac{x}{0.25}\ \ \Rightarrow\ \ u = \frac{x}{0.25}$

$$y\,(x) = 1.10100 + \frac{\left(\frac{x}{0.25}\right)}{1\,!}\,(0.7660625) + \frac{\left(\frac{x}{0.25}\right)\left[\left(\frac{x}{0.25}\right)+1\right]}{2\,!}\,(0.406375)$$
$$+ \frac{\left(\frac{x}{0.25}\right)\left[\left(\frac{x}{0.25}\right)+1\right]\left[\left(\frac{x}{0.25}\right)+2\right]}{3\,!}\,(0.09375)$$

$$= 1.10100 + \left(\frac{x}{0.25}\right)\,(0.7660625) + \left(\frac{x}{0.25}\right)\left(\frac{x+0.25}{0.25}\right)\left(\frac{0.406375}{2}\right) + \left(\frac{x}{0.25}\right)\left(\frac{x+0.25}{0.25}\right)\left(\frac{x+0.50}{0.25}\right)\left(\frac{0.09375}{6}\right)$$

$$= 1.10100 + x\left(\frac{0.7660625}{0.25}\right) + x\,(x+0.25)\left(\frac{0.406375}{2(0.25)(0.25)}\right) + x\,(x+0.25)(x+0.50)\left(\frac{0.09375}{2(0.25)(0.25)(0.50)}\right)$$

$$= 1.10100 + x\left(\frac{0.7660625}{0.25}\right) + \ (x^2 + 0.25\,x)\left(\frac{0.406375}{2(0.25)(0.25)}\right) + (x^2 + 0.25\,x)(x+0.50)\left(\frac{0.09375}{2(0.25)(0.25)(0.50)}\right)$$

$$= 1.10100 + x\left(\frac{0.7660625}{0.25}\right) + \ (x^2 + 0.25\,x)\left(\frac{0.406375}{2(0.25)(0.25)}\right)$$
$$+ (x^3 + 0.25\,x^2 + 0.50x^2 + 0.125\,x)\left(\frac{0.09375}{2(0.25)(0.25)(0.50)}\right)$$

To find $y(4)$:

$$f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$$

$$f(4) = 41$$

Verification : $f(2) = 2(2)^3 - 7(2)^2 + 6(2) + 1 \quad \Rightarrow \quad = 16 - 28 + 12 + 1$

$$f(2) = +1$$

Therefore the polynomial is correct.

6. From the following data, find the number of students whose weight is between 60 to 70.

weight in Lbs:	0 – 40	40 – 60	60 – 80	80 - 100	100 – 120
No. of Students	250	120	100	70	50

Solution : We form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40 (x_0)	250 (y_0)				
		120 (Δy_0)			
Below 60 (x_1)	370 (y_1)		**−20** $(\Delta^2 y_0)$		
		100 (Δy_1)		**−10** $(\Delta^3 y_0)$	
Below 80 (x_2)	470 (y_2)		−30 $(\Delta^2 y_1)$		**20** $(\Delta^4 y_0)$
		70 (Δy_2)		10 $(\boldsymbol{\Delta^3 y_1})$	
Below 100 (x_3)	540 (y_3)		−20 $(\Delta^2 y_2)$		
		50 (Δy_3)			
Below 120 (x_4)	590 (y_4)				

Now let us calculate the no. of students whose

The Newton's forward formula is

$$y\,(x) = y\,(x_0 + ph) = y_0 + \frac{u}{1\,!}\,\Delta\,y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\,y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\,y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\,y_0 + \cdots \ldots\ldots$$

$where \;\; u = \frac{x-x_0}{h}$. Here $x_0 = 40 \;\;\&\;\; h = 60 - 40 = 20 \quad (or) \quad 80 - 60 = 20$

$Let \quad u = \left(\frac{70-40}{20}\right) = \frac{30}{20} = 1.5$ Since $x = 70$

$$y\,(70) = 250 + \frac{(1.5)}{1\,!}\,(250) + \frac{(1.5)[1.5-1]}{2\,!}\,(-20) + \frac{(1.5)[1.5-1][1.5-2]}{3\,!}\,(-10) + \frac{(1.5)[1.5-1][1.5-2][1.5-3]}{4\,!}\,(-20) +$$

$$= 250 + (1.5)\,(250) + (1.5)[0.5]\left(-\frac{20}{2}\right) + (1.5)[0.5][-0.5]\left(-\frac{10}{6}\right) + (1.5)[0.5][-0.5][-1.5]\left(-\frac{20}{24}\right) +$$

$$= 250 + 180 - 7.5 + 0.625 + 0.46875$$

$$y\,(70) = 423.59 = 424\ \ (App)$$

7. The following data are taken from the steam table.

Temp $\,^0C$	140	150	160	170	180
$Pressure\ {}^{kgf}/_{cm^2}$	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature $t = 142^0$ & $t = 175^0$.

Solution : We form the difference table

t	p	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
140	3.685 (y_0)				
		1.169 (Δy_0)			
150	4.854 (y_1)		**0.279** $(\Delta^2 y_0)$		
		1.448 (Δy_1)		**0.047** $(\Delta^3 y_0)$	
160	6.302 (y_2)		0.326 $(\Delta^2 y_1)$		**0.002** $(\Delta^4$
		1.774 (Δy_2)		0.049 $(\Delta^3 y_1)$	
170	8.076 (y_3)		0.375 $(\Delta^2 y_2)$		
		2.149 (Δy_3)			
180	10.225 (y_4)				

To find the pressure at temperature $t = 142^0$

Let us use the Newton's forward formula :

$$y\,(x) = y\,(x_0 + ph) = y_0 + \frac{u}{1\,!}\,\Delta\,y_0 + \frac{u(u-1)}{2\,!}\,\Delta^2\,y_0 + \frac{u(u-1)(u-2)}{3\,!}\,\Delta^3\,y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4\,y_0 + \cdots \ldots\ldots$$

$where\ \ u = \frac{x-x_0}{h}$. Here $x_0 = 142$ & $h = 180 - 170 = 10\ \ (or)\ \ 160 - 150 = 10$

$Let\ \ u = \left(\frac{142-140}{10}\right) = \frac{2}{10} = 0.2$ Since $x = 142$

$$p\,(142) = 3.685 + \frac{(0.2)}{1\,!}\,(1.169) + \frac{(0.2)[0.2-1]}{2\,!}\,(0.279) + \frac{(0.2)[0.2-1][0.2-2]}{3\,!}\,(0.047) + \frac{(0.2)[0.2-1][0.2-2][0.2-3]}{4\,!}\,(0.002)$$

$$= 3.685 + (0.2)\,(1.169) + (0.2)[-0.8]\left(\frac{0.279}{2}\right) + (0.2)[-0.8][-1.8]\left(\frac{0.047}{6}\right) + (0.2)[-0.8][-1.8][-2.8]\left(\frac{0.002}{24}\right)$$

$$= 3.685 + 0.2338 \pm 0.02332 + 0.002256 \pm 0.0000672$$

$$p\,(142^0) = 3.898\,(App)$$

To find the pressure at temperature $t = 175^0$

Let us use the Newton's backward formula :

$$y\ (x) = y\ (x_n + ph)\ = y_n + \frac{V}{1\,!}\nabla\ y_n + \frac{V(V+1)}{2\,!}\ \nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\ \nabla^3 y_n + \cdots \ldots\ldots$$

$where\ \ u = \frac{x - x_n}{h}$. Here $x_n = 180\ \ \&\ \ h = 180 - 170 = 10\ \ (or)\ \ \ 160 - 150 = 10$

$$Let\ \ u = \left(\frac{175-180}{10}\right) = -\frac{5}{10}\ \ \Rightarrow\ \ u = -0.5$$

$$y\ (x) = 10.225 + \frac{(-0.5)}{1\,!}\ (2.149) + \frac{(-0.5)[(-0.5)+1]}{2\,!}\ (0.375) + \frac{(-0.5)[-0.5+1][-0.5+2]}{3\,!}\ (0.049) + \frac{(-0.5)[-0.5+1][-0.5+2][-0.5+3]}{4\,!}\ (0.002)$$

$$y\ (x) = 10.225 + (-0.5)\ (2.149) + (-0.5)[0.5]\ \left(\frac{0.375}{2}\right) + (-0.5)[0.5][1.5]\ \left(\frac{0.049}{6}\right) + (-0.5)[0.5][1.5][2.5]\ \left(\frac{0.002}{24}\right)$$

$$= 1.225 - 1.0745 - 0.0046875 - 0.0030625 - 0.000078125$$

$$p\ (175^0) = 9.100\ \ (App)$$