B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2010

Fourth Semester

Civil Engineering

MA2264 – NUMERICAL METHODS

(Regulation 2008)

(Common to Aeronautical Engineering and Electrical and Electronics Engineering)

Time : Three hours Maximum : 100 marks

Answer ALL Questions

PART A – (10 x 2 = 20 Marks)

1. Write sufficient condition for convergence of an iterative method for $f(x)=0$; written as $x=g(x)$.
2. Write down the procedure to find the numerically smallest eigen value of a matrix by power method.
3. Form the divided difference table for the data $(0,1),(1,4),(3,40)$ and $(4,85)$.
4. Define a cubic spline $S(x)$ which is commonly used for interpolation.
5. State the Romberg's integration formula with h_1 and h_2. Further, obtain the formula when $h_1 = h$ and $h_2 = h/2$.
6. Use two – point Gaussian quadrature formula to solve $\int_{-1}^{1} \frac{dx}{1+x^2}$.
7. Use Euler's method to find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = x+y,\ \ y(0)=1$.
8. Write the Adam – Bashforth predictor and corrector formulae.
9. Write down the explicit finite difference method for solving one dimensional wave equation.
10. Write down the standard five point formula to find the numerical solution of Laplace equation.

PART B – (5 x 16 = 80 marks)

11. (a) (i) Solve for a positive root of the equation $x^4 - x - 10 = 0$ using Newton – Raphson method.

(ii) Use Gauss – Seidal iterative method to obtain the solution of the equations: $9x - y + 2z = 9;\ x + 10y - 2z = 15;\ 2x - 2y - 13z = -17$.

Or

(b) (i) Find the inverse of the matrix by Gauss – Jordan method: $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.

(ii) Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

12. (a) (i) Use Lagrange's formula to find a polynomial which takes the values $f(0) = -12$, $f(1) = 0$, $f(3) = 6$ and $f(4) = 12$. Hence find $f(2)$.

(ii) Find the function $f(x)$ from the following table using Newton's divided difference formula:

x:	0	1	2	4	5	7
$f(x)$:	0	0	–12	0	600	7308

Or

(b) (i) If $f(0) = 1$, $f(1) = 2$, $f(2) = 33$ and $f(3) = 244$. find a cubic spline approximation, assuming M(0) = M(3) = 0. Also, find $f(2.5)$.

(ii) Given the following table, find the number of students whose weight is between 60 and 70 lbs:

Weight (in lbs) x:	0 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students:	250	120	100	70	50

13. (a) (i) Given the following data, find $y'(6)$ and the maximum value of y (if it exists)

x:	0	2	3	4	7	9
y:	4	26	58	112	466	922

(ii) Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{dxdy}{x+y}$ by trapezoidal formula by taking h = k = 0.1.

Or

(b) (i) Using Romberg's integration to evaluate $\int_0^1 \frac{dx}{1+x^2}$.

(ii) The velocity v of a particle at a distance S from a point on its path is given by the table below:

S (*meter*)	0	10	20	30	40	50	60
v (*m / sec*)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's 1/3rd rule and Simpson's 3/8th rule.

14. (a) (i) Use Tailor series method to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$, correct to 4 decimal accuracy.

(ii) Use Milne's predictor – corrector formula to find $y(0.4)$, given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, $y(0) = 1,\ y(0.1) = 1.06,\ y(0.2) = 1.12$ and $y(0.3) = 1.21$.

Or

(b) (i) Use Rung – Kutta method of fourth order to find $y(0.2)$, given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, taking h = 0.2.

(ii) Given $\frac{dy}{dx} = xy + y^2,\ y(0) = 1,\ y(0.1) = 1.1169,\ y(0.2) = 1.2774,\ y(0.3) = 1.5041$. Use Adam's method to estimate $y(0.4)$.

15. (a) Deduce the standard five point formula for $\nabla^2 u = 0$. Hence, solve it over the square region given by the boundary conditions as in the figure below:

	u_1	u_2
	u_3	u_4

(ii) Obtain the Crank – Nicholson finite difference method by taking $\lambda = \frac{kc^2}{h^2} = 1$. Hence, find $u(x,t)$ in the rod for two times steps for the heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, given $u(x,0) = \sin(\pi x),\ u(0,t) = 0,\ u(1,t) = 0$. Take h = 0.2.