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Degree / Branch: B.Tech, IV Information Technology
Subject: Discrete Mathematics
Unit /Title : I & Propositional Calculus

Part A
2 Marks Questions

1. What is proposition?
   Solution:
   A Proposition is a declarative sentence that is either true or false, but not both.
   Eg: $2 > 1\ [True]$
   $$1 + 7 = 9\ [False]$$

2. What is atomic statement? Give an example.
   Solution:
   Declarative sentences which cannot be further split into simpler sentences are called atomic statements.
   Eg: Ram is a boy

3. What is compound statement? Give an example.
   Solution:
   Declarative sentences which can be further split into simpler sentences are called compound statement. Compound statements are constructed by combining the connectives 'and', 'or' , 'but' , etc.,

4. Write the truth table for negation?
   Solution:
   The negation of a statement is generally formed by introducing the word 'not' at a proper place in the statement.
   Truth table for negation

P	¬P
T	F
F	T

5. Without using table prove the following
   $P \wedge ((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \Leftrightarrow R$
   Solution:
   $$\begin{aligned} P \wedge ((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) &\Leftrightarrow P \wedge (\neg P \wedge (Q \vee \neg Q)) \\ &\Leftrightarrow P \wedge (\neg P \wedge T) \\ &\Leftrightarrow P \wedge \neg P \\ &\Leftrightarrow F \\ &\Leftrightarrow \text{R} \end{aligned}$$

6. Express the statement “Good food is not cheap” in symbolic form.
Solution:
P: food is good.
Q: food is cheap
Symbolic form: $P \rightarrow \neg Q$

7. Obtain PDNF for $\neg P \vee Q$
Solution:

$$\neg P \vee Q \Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee ((P \vee \neg P) \wedge Q)$$
$$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$$
$$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \text{ which is PDNF}$$

8. Write an equivalent formula for $P \wedge (Q \leftrightarrow R)$ which contains neither the biconditional nor the conditional.
Solution:

$$P \wedge (Q \leftrightarrow R) \Leftrightarrow P \wedge ((Q \rightarrow R) \wedge (R \rightarrow Q))$$
$$\Leftrightarrow P \wedge ((\neg Q \vee R) \wedge (\neg R \vee Q))$$

9. Write an equivalent formula for $P \rightarrow (Q \rightarrow R)$
Solution:
$P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow \neg P \vee (\neg Q \vee R)$

10 . Show that the propostion $(P \vee Q) \leftrightarrow (Q \vee P)$ is a tautology
Solution:

P	Q	$P \vee Q$	$Q \vee P$	$\neg(P \vee Q)$	$\neg(Q \vee P)$	$(P \vee Q) \leftrightarrow (Q \vee P)$
T	T	T	T	F	F	T
T	F	T	T	F	F	T
F	T	T	T	F	F	T
F	F	F	F	T	T	T

The last column contains only T.
Given proposition is tautology.

11. Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \vee \neg Q)$ is a tautology.
Solution:

P	Q	$\neg Q$	$\neg P$	$P \wedge \neg Q$	$\neg P \vee \neg Q$	$Q \vee (P \wedge \neg Q) \vee (\neg P \vee \neg Q)$
T	T	F	T	F	T	T
T	F	T	T	T	T	T
F	T	F	F	F	F	T
F	F	T	F	F	T	T

The last column contains only T.
Given statement is tautology.

12. Using the truth table verify $(P \wedge Q) \wedge (\neg(P V Q))$ is contradiction.

Solution:

P	**Q**	**P∧Q**	**P∨Q**	**¬ (P∨Q)**	**(P∧Q) ∧ (¬ (P∨Q))**
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

The last column contains only F
Given statement is contradiction.

13. Define contrapositive.

Solution:

If $P \rightarrow Q$ is an implication, then the converse of $P \rightarrow Q$ is the implication $Q \rightarrow P$ and the contrapositive of $P \rightarrow Q$ is the implication $\neg Q \rightarrow \neg P$

14. Give the converse and the contrapositive of the implication "If it is raining, then I get wet"

Solution: P: It is raining
Q: I get wet.
$Q \rightarrow P$ (Converse) If I get wet then it is raining
$\neg Q \rightarrow \neg P$(Contrapositive): If I do not get wet, then it is not raining.

15. Define the term "Logically equivalent"

Solution:

The propositions P and Q are called logically equivalent if $P \rightarrow Q$ is a tautology.
It is denoted by $P \equiv Q$

16. Write the Statement " The crop will be destroyed if there is a flood" in symbolic form

Solution: P: Crop will be destroyed
Q: There is a flood
Symbolic form: $Q \rightarrow P$

17. State and prove Duality principle theorem

Solution:

If $A \text{ and } A^*$ be dual formulas and if $p_1, p_2, \dots p_n$ be simple variables that occur in $A \text{ and } A^*$
ie) $A = A(p_1, p_2, \dots p_n)$ and $A^* = A^*(p_1, p_2, \dots p_n)$ then

$$\neg A(p_1, p_2, \dots p_n) \Leftrightarrow A^*(p_1, p_2, \dots p_n) \text{and}$$

$$A(\neg p_1, \neg p_2, \dots \neg p_n) \Leftrightarrow \neg A^*(p_1, p_2, \dots p_n)$$

That is the negation of a formula is equivalent to its dual in which every variable is replaced by its negation.

18. Define functionally complete sets of connectives.
Any set of connectives in which every formula can be expressed as another equivalent formula containing connectives from this set is called functionally complete set of connective.
Eg: The set of connectives $\{\wedge, \vee\}$

19. Prove that $\{\neg, \vee\}$ is a functionally complete set of connectives.
Solution:
It is enough to show that all formulas with other connectives, there exists a equivalent formula which contains $\neg$ and $\vee$ only.
Eg: i) $P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

ii) $P \to Q \Leftrightarrow \neg P \vee Q$

iii) $P \wedge Q \Leftrightarrow \neg(\neg P \vee \neg Q)$

Hence $\{\neg, \vee\}$ is functionally complete set of connectives.

20. Show that $\{\wedge, \vee\}$ is not functionally complete.
Solution:
$\neg P$ cannot be expressed using the connectives $\{\wedge, \vee\}$. Since no such contribution of statement exist with $\{\wedge, \vee\}$ as input is T and the output is F.

21. Construct the truth table for NAND
Solution:

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

22. obtain disjunctive normal forms of $P \wedge (P \to Q)$
Let $S \equiv P \wedge (P \to Q)$

$\Leftrightarrow P \wedge (\neg P \vee Q)$

$\Leftrightarrow (P \wedge \neg P) \vee (P \wedge Q)$ which is DNF

23. Obtain a CNF for $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R)$
Solution: $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$

$\Leftrightarrow ((\neg P V (Q \wedge R)) \wedge (P \vee (\neg Q \wedge \neg R)$

$\Leftrightarrow ((\neg P \vee Q) \wedge (\neg P \vee R)) \wedge ((P \vee \neg Q) \wedge (P \vee \neg R))$ (Distributive law)

This is CNF, as it is a product of elementary sums.

24. Define Valid argument
If a conclusion is derived from a set of premised by using the accepted rules of reasoning, then such a process of derivation is called a deduction or a formal proof and the argument or conclusion is called a valid argument or valid conclusion.

25. Determine whether the conclusion c follows logically from the premised H_1, H_2 and H_3:
$H_1: P \to Q,\ H_2: P,\ H_3: Q$. Are given premises valid?
Solution:

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$((P \to Q) \wedge P) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The given premises are valid.

26. Write the rules for inference theory.
Rule P: A Premise may be introduced at any point in the derivation.
Rule T: A formula S may be introduced in a derivation if S is a tautologically implied by any one or more of the preceding formulas in the derivation.
Rule CP: If we can derive S from R and a set of premised, then we can derive $R \to S$ from the set of premises alone.

27. Demonstrate that R is valid inference from the premises $P \to Q, Q \to R\ and\ P$
Solution:
$i)\ \ P \to Q \quad Rule\ P$
$ii)\ \ P \quad Rule\ P$
$iii)\ \ Q \quad Rule\ T\ From\ (i),(ii)\ and\ P, P \to Q\ \Rightarrow Q$
$iv)\ Q \to R \quad Rule\ P$
$V)\ \ R \quad Rule\ T\ \ From\ (iii),(iv)\ and\ Q, Q \to R\ \Rightarrow R$

28. Show that the following sets of premises are inconsistent.
$P \to Q\ , P \to R, Q \to \neg R, P$
Solution:
$i)\ \ P \to Q \quad Rule\ P$
$ii)\ \ Q \to \neg R \quad Rule\ P$
$iii)\ P \to \neg R \quad Rule\ T\ From\ (i),(ii)\ and\ P \to Q, Q \to R \Rightarrow\ \ P \to R$
$iv)\ \ P \quad Rule\ P$
$v\ )\ \ \neg R \quad Rule\ T\ From\ (iii),(iv)$

$vi)\ \ P \rightarrow R \quad Rule\ P$
$vii)\ R \qquad Rule\ T\ \ From\ (iv),(vi),P,P \rightarrow R\ \Rightarrow\ \ R$
$viii) R\wedge\neg R \quad Rule\ T$
Given premises are inconsistent

29. If premises $P, Q\ and\ R$ are inconsistent, prove that $\neg R$ is a conclusion from $P\ and\ Q$.
Solution:
Given $P, Q\ and\ R$ are inconsistent, $P\wedge Q\wedge R \Rightarrow F$ where F is contradiction
To prove: $P\wedge Q \Rightarrow \neg R$
Assume $P\wedge Q$ is true
If $\neg R$ is false $\Rightarrow$ R is true
Then only $P\wedge Q\wedge R$ is true which is contradiction.
$\neg R$ is true.
Hence $P\wedge Q \Rightarrow \neg R$

30. What is duality law of logical expression? Give the dual of $(P\vee F)\wedge(Q\vee T)$.
Solution:
In an expression, if we replace $\vee, \wedge, T, F$ respectively by $\wedge, \vee, F, T$. The resulting new formula is the dual of the given expression.
Dual of given formula is $(P\wedge T)\vee(Q\wedge F)$.

Part B

31. i) Without using truth table, show that $(\neg P\wedge(\neg Q\wedge R))\vee(Q\wedge R)\vee(P\wedge R) \Leftrightarrow\ R$

Solution:
$(\neg P\wedge(\neg Q\wedge R))\vee(Q\wedge R)\vee(P\wedge R)$
$\Leftrightarrow \big(\neg P\wedge(\neg Q\wedge R)\big)\vee\big((Q\vee P)\wedge R\big) \qquad [Distributive\ law]$
$\Leftrightarrow \big((\neg P\wedge\neg Q)\wedge R\big)\vee\big((Q\vee P)\wedge R\big) \qquad [Associative\ law]$
$\Leftrightarrow (\neg(P\vee Q)\wedge R)\vee\big((Q\vee P)\wedge R\big) \qquad [De\ morgan's\ law]$
$\Leftrightarrow (\neg(P\vee Q)\wedge R)\vee\big((P\vee Q)\wedge R\big) \qquad [commutative\ law]$
$\Leftrightarrow \big(\neg(P\vee Q)\vee (P\vee Q)\big)\wedge R \qquad [commutative\ law]$
$\Leftrightarrow T\wedge R \qquad \big[\neg P\wedge P \Rightarrow\ T\big]$
$\Leftrightarrow R \qquad \big[T\wedge P\ \Rightarrow\ P\big]$

ii) Without using truth table, show that $(P\vee Q)\wedge(\neg P\wedge(\neg P\wedge Q)) \Leftrightarrow (\neg P\wedge Q)$

Solution:
$(P\vee Q)\wedge(\neg P\wedge(\neg P\wedge Q))$
$\Leftrightarrow (P\vee Q)\wedge((\neg P\wedge\neg P)\wedge Q) \qquad [Associative\ law]$

$\Leftrightarrow (P \vee Q) \wedge (\neg P \wedge Q) \qquad [Idempotent\ law]$

$\Leftrightarrow (P \wedge (\neg P \wedge Q)) \vee (Q \wedge (\neg P \wedge Q)) \qquad [Distributive\ law]$

$\Leftrightarrow ((P \wedge \neg P) \wedge Q) \vee ((Q \wedge \neg P) \wedge Q) \qquad [Associative\ law]$

$\Leftrightarrow \big((P \wedge \neg P) \vee (Q \wedge \neg P)\big) \wedge Q \qquad [Distributive\ law]$

$\Leftrightarrow \big(F \vee (Q \wedge \neg P)\big) \wedge Q \qquad \big[P \wedge \neg P \Rightarrow F\big]$

$\Leftrightarrow (Q \wedge \neg P) \wedge Q \qquad \big[F \vee P \Rightarrow P\big]$

$\Leftrightarrow (\neg P \wedge Q) \wedge Q \qquad [Commutative\ law]$

$\Leftrightarrow \neg P \wedge (Q \wedge Q) \qquad [Associative\ law]$

$\Leftrightarrow \neg P \wedge Q \qquad [Idempotent\ law]$

32. i) Without using truth table obtain disjunctive normal forms of
$\neg(P \vee Q) \leftrightarrow (P \wedge Q)$

$\equiv \big(\neg(P \vee Q) \rightarrow (P \wedge Q)\big) \wedge \big((P \wedge Q) \rightarrow \neg(P \vee Q)\big)$
$\equiv (\neg\neg(P \vee Q) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee \neg(P \vee Q))$
$\equiv ((P \vee Q) \vee (P \wedge Q)) \wedge ((\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q))$
$\equiv \big(((P \vee Q) \vee P) \wedge ((P \vee Q) \vee Q)\big) \wedge ((\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q))$
$\equiv ((P \vee Q) \wedge (P \vee Q)) \wedge ((\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q))$
$\equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$
$\equiv ((P \vee Q) \wedge \neg P) \vee ((P \vee Q) \wedge \neg Q)$
$\equiv (P \wedge \neg P) \vee (Q \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg Q)$
$\equiv F \vee (Q \wedge \neg P) \vee (P \wedge \neg Q) \vee F$
$(Q \wedge \neg P) \vee (P \wedge \neg Q)$

ii) Without using truth table obtain conjunctive normal forms of
$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
$\equiv P \rightarrow ((\neg P \vee Q) \wedge \neg(\neg Q \vee \neg P))$
$\equiv P \rightarrow ((\neg P \vee Q) \wedge (Q \wedge P))$
$\equiv P \rightarrow (\neg P \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))$
$\equiv P \rightarrow (\neg P \wedge (P \wedge Q)) \vee (Q \wedge (Q \wedge P))$
$\equiv P \rightarrow ((\neg P \wedge P) \wedge Q) \vee ((Q \wedge Q) \wedge P)$
$\equiv P \rightarrow F \vee (Q \wedge P)$
$\equiv P \rightarrow (Q \wedge P)$
$\equiv \neg P \vee (Q \wedge P)$
$\equiv (\neg P \vee Q) \wedge (\neg P \vee P)$
$\equiv (\neg P \vee Q) \wedge T$
$\equiv \neg P \vee Q$ which is CNF

33. i) Without constructing the truth table obtain the product of sums canonical form of the formula $(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$

Solution:

$(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$

$\equiv (\neg\neg P \vee R) \wedge (Q \rightarrow R) \wedge (R \rightarrow Q)$

$\equiv (P \vee R) \wedge (Q \rightarrow R) \wedge (R \rightarrow Q)$

$\equiv (P \vee R) \wedge (\neg Q \vee R) \wedge (\neg R \vee Q)$

$\equiv (P \vee R \vee (Q \wedge \neg Q)) \wedge ((P \wedge \neg P) \vee \neg Q \vee R) \wedge ((P \wedge \neg P) \vee \neg R \vee Q)$

$\equiv (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$
$\wedge (P \vee \neg R \vee Q) \wedge (\neg P \vee \neg R \vee Q)$

$\equiv (P \vee R \vee Q) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$
$\wedge (P \vee \neg R \vee Q) \wedge (\neg P \vee \neg R \vee Q)$ writing the repeating terms only once which is the PCNF of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$

ii) Without using truth table obtain the product of sums canonical form of $(P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

Solution:

$Let\ S \equiv (P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$ which is PDNF

$\neg S\ represents\ the\ missing\ terms\ in\ PDNF$

$\neg S \equiv (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$
$\vee (P \wedge \neg Q \wedge \neg R)$

$\neg\neg S \equiv \neg(P \wedge \neg Q \wedge R) \wedge \neg(P \wedge Q \wedge \neg R) \wedge \neg(\neg P \wedge \neg Q \wedge R) \wedge \neg(\neg P \wedge Q \wedge \neg R)$
$\wedge \neg(P \wedge \neg Q \wedge \neg R)$

$S \equiv (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R)$
$\wedge (\neg P \vee Q \vee R)$ which is PCNF

34. Without constructing the truth table obtain PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$. Also find PCNF.

Solution:

$Let\ S \equiv (P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$\equiv ((P \wedge Q) \wedge (R \vee \neg R)) \vee (\neg P \wedge (Q \vee \neg Q) \wedge R) \vee ((P \vee \neg P) \wedge (Q \wedge R))$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee \left(((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \wedge R\right)$
$\vee (P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$
$\vee (P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

which is PDNF

$\neg S\ represents\ the\ missing\ terms\ in\ PDNF$

$\neg S \equiv (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

$\neg\neg S \equiv \neg(P \wedge \neg Q \wedge R) \wedge \neg(P \wedge \neg Q \wedge \neg R) \wedge \neg(\neg P \wedge Q \wedge \neg R) \wedge \neg(\neg P \wedge \neg Q \wedge \neg R)$

$S \equiv (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

which is PCNF

35. i) Without constructing the truth table show that

$$\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$$

Solution:

$\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q))$

$\Leftrightarrow \neg(P \wedge Q) \rightarrow ((\neg P \vee \neg P) \vee Q)$

$\Leftrightarrow \neg(P \wedge Q) \rightarrow (\neg P \vee Q)$

$\Leftrightarrow \neg\neg(P \wedge Q) \vee (\neg P \vee Q)$

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$

$\Leftrightarrow (P \vee (\neg P \vee Q)) \wedge (Q \vee (\neg P \vee Q))$

$\Leftrightarrow ((P \vee \neg P) \vee Q) \wedge (Q \vee (\neg P \vee Q))$

$\Leftrightarrow (T \vee Q) \wedge (Q \vee (\neg P \vee Q))$

$\Leftrightarrow T \wedge (Q \vee (\neg P \vee Q))$

$\Leftrightarrow Q \vee (\neg P \vee Q)$

$\Leftrightarrow (\neg P \vee Q) \vee Q$

$\Leftrightarrow \neg P \vee (Q \vee Q)$

$\Leftrightarrow \neg P \vee Q$

ii) Without constructing the truth table show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology

$(P \wedge Q) \rightarrow (P \vee Q)$

$\Rightarrow \neg(P \wedge Q) \vee (P \vee Q)$

$\Rightarrow (\neg P \vee \neg Q) \vee (P \vee Q)$

$\Rightarrow \neg P \vee (\neg Q \vee P) \vee Q$

$\Rightarrow \neg P \vee (P \vee \neg Q) \vee Q$

$\Rightarrow (\neg P \vee P) \vee (\neg Q \vee Q)$

$\Rightarrow T \vee T$

$\Rightarrow T$

$(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology

36. Without constructing the truth table obtain the PDNF of $\neg P \vee Q$. Also find PCNF.

Solution:

Let $S \equiv \neg P \vee Q$

$\equiv \big(\neg P \wedge (Q \vee \neg Q)\big) \vee \big((P \vee \neg P) \wedge Q\big)$

$\equiv (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$

$\equiv (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$ *is a* PDNF

$\neg S \equiv P \wedge \neg Q$

$\neg\neg S \equiv \neg (P \wedge \neg Q)$

$S \equiv \neg P \vee Q$ is a PCNF

37. i) Without constructing the truth table show that $R \vee S$ from the following premises, $C \vee D, (C \vee D) \rightarrow \neg H, \neg H \rightarrow (A \wedge \neg B)$ *and* $(A \wedge \neg B) \rightarrow (R \vee S)$.

Solution:
$i)\ C \vee D$ $\quad Rule\ P$
$ii)\ (C \vee D) \rightarrow \neg H$ $\quad Rule\ P$
$iii)\ \neg H \rightarrow (A \wedge \neg B)$ $\quad Rule\ P$
$iv)\ (C \vee D) \rightarrow (A \wedge \neg B)$ $\quad Rule\ T, ii\ , iii\ and\ hypothetical\ syllogism$
$v)\ (A \wedge \neg B) \rightarrow (R \vee S)$ $\quad Rule\ P$
$vi)\ (C \vee D) \rightarrow (R \vee S)$ $\quad Rule\ T, iv\ , v\ and\ hypothetical\ syllogism$
$vii)\ R \vee S$ $\quad Rule\ T, i, vi\ and\ modus\ ponens$

ii) Without constructing the truth table show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M\ and\ \neg M.$
$i)\ P \vee Q$ $\quad Rule\ P$
$ii)\ P \rightarrow M$ $\quad Rule\ P$
$iii)\ \neg M$ $\quad Rule\ P$
$iv)\ \neg P$ $\quad Rule\ T, ii, iii\ and\ modus\ tollens$
$v)\ Q$ $\quad Rule\ T, i, iv\ and\ disjunctive\ syllogism$
$vi)\ Q \rightarrow R$ $\quad Rule\ P$
$vii)\ R$ $\quad Rule\ T, v, vi\ and\ modus\ ponens$
$viii)\ R \wedge (P \vee Q)$ $\quad Rule\ T, vii, i\ and\ conjuction$

38. Show that the following premises are inconsistent.
    a. If Jack misses many classes through illness, then he fails high school.
    b. If Jack fails high school, then he is uneducated
    c. If Jack reads a lot of books, then he is not uneducated.
    d. Jack misses many classes through illness and reads a lot of books.

Solution:
Let C represents Jack misses many classes through illness
Let F represents Jack fails high school
Let E represents Jack is uneducated
Let B represents Jack reads lot of books
The symbolic representation of the problem is
$C \rightarrow F, F \rightarrow E, B \rightarrow \neg E, C \wedge B$ are inconsistent.

$i)\ C \wedge B$ $\quad Rule\ \ P$
$ii)\ C$ $\quad Rule\ T, i, and\ Simplification$
$iii)\ B$ $\quad Rule\ T, i\ and\ Simplification$
$iv)\ C \rightarrow F$ $\quad Rule\ P$
$v)\ F \rightarrow E$ $\quad Rule\ P$
$vi)\ C \rightarrow E$ $\quad Rule\ T, iv, v\ and\ hypothetical\ syllogism$
$vii)\ B \rightarrow \neg E$ $\quad Rule\ p$
$viii)\ \neg E$ $\quad Rule\ T, iii, vii\ and\ modus\ ponens$
$ix)\ E$ $\quad Rule\ T, ii, vi\ and\ modus\ ponens$

$x)\ E \land \neg E \quad Rule\ T, viii, xi\ and\ conjuction$

$xi)\ F \quad Rule\ T, x\ and\ negation\ law$

The set of given premises are inconsistent.

39. i) Show that the following set of premises is inconsistent.

If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and bank will loan him money.

Solution:

Let C represents the contract is valid

Let P represents John is liable for penalty

Let B represents John will go bankrupt

Let M represents bank will loan him money

The symbolic representation of the problem is

$C \rightarrow P, P \rightarrow B, M \rightarrow \neg B, C \land M$ are inconsistent.

$i)\ C \land M \quad Rule\ \ P$

$ii)\ C \quad Rule\ T, i, and\ Simplification$

$iii)\ M \quad Rule\ T, i, and\ Simplification$

$iv)\ C \rightarrow P \quad Rule\ P$

$v)\ P \rightarrow B \quad Rule\ P$

$vi)\ C \rightarrow B \quad Rule\ T, iv, v\ and\ hypothetical\ syllogism$

$vii)\ M \rightarrow \neg B\ Rule\ p$

$viii)\ \neg B \quad Rule\ T, iii, vii\ and\ modus\ ponens$

$ix)\ B \quad Rule\ T, ii, vi\ and\ modus\ ponens$

$x)\ B \land \neg B \quad Rule\ T, viii, xi\ and\ conjuction$

$xi)\ F \quad Rule\ T, x\ and\ negation\ law$

The set of given premises are inconsistent.

ii) By Indirect proof, Show that $P \rightarrow Q, Q \rightarrow R, P \lor R \Rightarrow R$.

Solution:

$Let\ us\ assume\ that\ \neg R\ be\ the\ additional\ premises\ and$
$prove\ a\ contradiction$

$i)\ \neg R \quad Rule\ Additional\ P$

$ii)\ Q \rightarrow R \quad Rule\ P$

$iii)\ \neg Q \quad Rule\ T, i, ii\ and\ modus\ tollens$

$iv)\ P \rightarrow Q \quad Rule\ P$

$v)\ \neg P \quad Rule\ T, iii, iv\ and\ modus\ tollens$

$vi)\ \neg P \land \neg R \quad Rule\ T, iii, v\ and\ conjuction$

$vii)\ \neg(P \lor R) \quad Rule\ T, vi\ and\ Demorgans\ law$

$viii)\ P \lor R \quad Rule\ P$

$ix)\ \neg(P \lor R) \land (P \lor R) \quad Rule\ T, vii, viii\ and\ conjuction$

$x)\ F \quad Rule\ T, ix\ and\ \ Negation$

40. i) Without using truth tables, show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is tautology.

Solution:

$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

$$\Rightarrow \big((Q \vee P) \wedge (Q \vee \neg Q)\big) \vee (\neg P \wedge \neg Q) \quad [Distributive\ law]$$

$$\Rightarrow \big((Q \vee P) \wedge T\big) \vee (\neg P \wedge \neg Q) \quad [Q \vee \neg Q = T]$$

$$\Rightarrow (Q \vee P) \vee (\neg P \wedge \neg Q) \quad [Q \wedge T = Q]$$

$$\Rightarrow \big((Q \vee P) \vee \neg P\big) \wedge \big((Q \vee P) \vee \neg Q\big) \quad [Distributive\ law]$$

$$\Rightarrow \big(Q \vee (P \vee \neg P)\big) \wedge \big((P \vee Q) \vee \neg Q\big) \quad [Associative\ \&\ commutative\ law]$$

$$\Rightarrow \big(Q \vee (P \vee \neg P)\big) \wedge \big(P \vee (Q \vee \neg Q)\big) \quad [Associative\ law]$$

$$\Rightarrow (Q \vee T) \wedge (P \vee T) \quad [Q \vee \neg Q = T]$$

$$\Rightarrow T \wedge T \quad [Q \vee T = T]$$

$$\Rightarrow T$$

$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is tautology

ii) Without constructing the truth table show that S is valid inference from the premises $P \rightarrow \neg Q, Q \vee R, \neg S \rightarrow P\ and\ \neg R.$

Solution:

$i)\ Q \vee R \qquad Rule\ P$

$ii)\ \neg R \qquad Rule\ P$

$iii)\ Q \qquad Rule\ T, i, ii\ and\ dusjunction\ syllogism$

$iv)\ P \rightarrow \neg Q \qquad Rule\ P$

$v)\ \neg P \qquad Rule\ T, iii, iv\ and\ Modus\ tollens$

$vi)\ \neg S \rightarrow P \qquad Rule\ P$

$vii)\ \neg\neg S \qquad Rule\ T, v, vi\ and\ Modus\ tollens$

$viii)\ S \qquad Rule\ T, vii\ and\ Negation$