

## Propositional Calculus

### Demorgan's laws

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

### Commutative laws

$$P \wedge Q \equiv Q \wedge P$$

$$P \vee Q \equiv Q \vee P$$

### Associative laws

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

### Distributive laws

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

### Implication

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$$

### Other important laws

$$\neg\neg P \equiv P$$

$$\neg P \vee P \equiv T$$

$$\neg P \wedge P \equiv F$$

$$P \vee T \equiv T$$

$$P \wedge T \equiv P$$

$$P \vee F \equiv P$$

$$P \wedge F \equiv F$$

### Simplification

$$P \wedge Q \Rightarrow P \text{ or } Q$$

### Addition

$$P \text{ or } Q \Rightarrow P \vee Q$$

### Hypothetical Syllogism

$$(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$$

### Modus Ponens

$$(P \rightarrow Q) \wedge P \Rightarrow Q$$

### Modus Tollens

$$(P \rightarrow Q) \wedge \neg Q \Rightarrow \neg P \text{ or } (P \rightarrow \neg Q) \wedge Q \Rightarrow \neg P$$

### Disjunctive Syllogism

$$(P \vee Q) \wedge \neg P \Rightarrow Q \text{ or } (P \vee Q) \wedge \neg Q \Rightarrow P$$