C 3276

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Sixth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1251/MA 1011 — NUMERICAL METHODS

(Common to Chemical Engineering/Information Technology/Electronics and Communication Engineering/Mechanical Engineering)

Time : Three hours | Maximum : 100 marks

Answer **ALL** questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the formula for the method of false position to determine a root of $f(x) = 0$.

2. State the sufficient condition on $\phi(x)$ for the convergence of an iterative method for $f(x) = 0$ written as $x = \phi(x)$.

3. Show that the divided differences are symmetrical in their arguments.

4. State Newton's backward difference interpolation formula.

5. State Romberg's method integration formula to find the value of $I = \int_a^b f(x)dx$, using h and $\frac{h}{2}$.

6. Write down the Simpson's $-\frac{3}{8}$ rule of integration given $(n+1)$ data.

7. State the Taylor series formula to find $y(x_1)$ for solving $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

8. State Milne's predictor-corrector formula.

9. Name at least two numerical methods that are used to solve one dimensional diffusion equation.

10. Write down Laplace equation and its finite difference analogue and the standard five-point formula.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove the quadratic convergence of Newton-Raphson method. Find a positive root of $f(x) = x^3 - 5x + 3 = 0$, using this method.

(ii) Solve the following system by Gauss-Seidal method :

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35.$$

Or

(b) (i) Find the inverse of the matrix by Gauss-Jordan method :

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$$

(ii) Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

12. (a) (i) If $f(0) = 0, f(1) = 0, f(2) = -12, f(4) = 0, f(5) = 600$ and $f(7) = 7308$, find a polynomial that satisfies this data using Newton's divided difference interpolation formula. Hence, find $f(6)$.

(ii) Given the following table, find $f(2.5)$ using cubic spline functions :

i	0	1	2	3
x_i	1	2	3	4
$f(x_i)$	0.5	0.3333	0.25	0.2

Or

(b) (i) Find the Lagrange's polynomial of degree 3 to fit the data :

$y(0) = -12, y(1) = 0, y(3) = 6$ and $y(4) = 12$. Hence, find $y(2)$.

(ii) Find a polynomial of degree two for the data by Newton's forward difference method :

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

13. (a) (i) Find $f'(6)$ and the maximum value of $y = f(x)$ given the data :

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	992

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg method by taking $h = 0.5, 0.25$ and 0.125 successively.

Or

(b) (i) Evaluate $I = \int_0^1 \frac{dx}{1+x}$ using three point Gauss-quadrature formula.

(ii) Use Trapezoidal rule to evaluate $I = \int_1^2 \int_1^2 \frac{dxdy}{x+y}$ taking 4 subintervals.

14. (a) (i) Solve $\frac{dy}{dx} = \log_{10}(x+y)$, $y(0) = 2$ by Euler's modified method and find the values of $y(0.2), y(0.4)$ and $y(0.6)$, taking $h = 0.2$.

(ii) Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, find the value of $y(0.1)$ by Runge-Kutta method of fourth order.

Or

(b) (i) Solve $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, using Milne's predictor-corrector formulae and find $y(0.4)$. Use Taylor series method to find $y(0.1)$, $y(0.2)$ and $y(0.3)$.

(ii) Compute $y(0.2)$, given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, by Runge-Kutta method of fourth order, taking $h = 0.2$.

15. (a) (i) Solve the boundary value problem $y'' = xy$, subject to the conditions $y(0) + y'(0) = 1$, $y(1) = 1$, taking $h = \frac{1}{3}$, by finite difference method.

(ii) Using Bender-Schmitt formula, solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, u(0,t) = 0$, $u(5,t) = 0, u(x,0) = x^2(25 - x^2)$. Assume $\Delta x = 1$. Find $u(x,t)$ up to $t = 5$.

Or

(b) (i) Use Crank-Nicholson scheme to solve $\frac{\partial^2 u}{\partial x^2} = 16\frac{\partial u}{\partial t}$, $0 < x < 1$ and $t > 0$ given $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = 100t$. Compute $u(x,t)$ for one time step, taking $\Delta x = \frac{1}{4}$.

(ii) Evaluate $u(x,t)$ at the pivotal points of the equation $16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $u(0,t) = 0, u(5,t) = 0, \frac{\partial u}{\partial t}(x,0) = 0$ and $u(x,0) = x^2(5 - x)$ taking $\Delta x = 1$ and up to $t = 1.25$.