L 1118

B.E./B.Tech. DEGREE **EXAMINATION,** MAY/JUNE 2006.

Fourth Semester

Civil Engineering

MA 1251 — NUMERICAL METHODS

(Common to **Aeronautical** Engg., Electrical and Electronics **Engg., Mechatronics Engg. and Metallurgical Engg. and Common** to B.E. (Part-time) Third Semester)

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer **ALL** questions.

PART A — (10 x 2 = 20 marks)

1. If $f(x) = 0$ has **root** between $x = a$ and $x = b$, then write the first approximate root by the method of false position.

2. **Solve** the system of equations $x - 2y = 0, 2x + y = 5$ **by Gaussian** elimination method.

3. Obtain the divided difference table **for** the following **data**

x :	2	3	5
y :	0	14	102

4. Find a polynomial for the following **data by** Newton's **backward** difference formula

x :	0	1	2	3
$f(x)$:	−3	2	9	18

5. Write down the **expressions for** $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **at** $x = x_0$ by Newton's **forward difference formula.**

6. Evaluate $\int_1^4 f(x)\,dx$ from the table by Simpson's 3/8 rule

x :	1	2	3	4
$f(x)$:	1	8	27	64

7. State Adarns-Bashforth predictor and corrector formula.

8. Find $y(1.1)$, given $\frac{dy}{dx} = x + y$, $y(1) = 2$ by Euler's method.

9. **Write down the** finite difference scheme for the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 2.$$

10, **State the** implicit finite **difference** scheme **for** one dimensional heat **equation.**

PART B — (5 × 16 = 80 marks)

11. (i) **Using** Lagrange's interpolation formula **fit a polynomial** *to* the following data :

x :	-1	0	2	3
y :	-8	3	1	12

and hence find y **at** $x = 1.5$ (8)

(ii) Fit the cubic spline for the data :

x :	1	2	3
y :	-6	-1	16

Hence evaluate $y(1.5)$. (8)

12. (a) (i) Find an iterative **formula to** find $\sqrt{N}$ **where** N **is a positive integer** using **Newton's** method **and hence** find $\sqrt{11}$. (8)

(ii) Solve the following system **of** equations by Gauss–Seidel method **correct** to **three decimal places.**

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72.$$

(8)

Or

(b) (i) Find **the inverse** of the **given matrix** by Gauss–Jordon method $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ (8)

(ii) Find the numerically largest eigenvalue of **A** $= \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by power method and the corresponding **eigenvector** (correct to three decimal points). Start with initial eigenvector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. (8)

13. (a) (i) Compute $\int_0 \frac{dx}{1+x^2}$ by using **Trapezoidal** rule, taking $h = 0.5$ and $h = 0.25$. Hence find the value **of** the above integration by Romberg's method. (8)

(ii) Evaluate $I = \int_0^1 \frac{dt}{1+t}$ by using three point Gaussian **quadrature formula.** (8)

Or

(b) Evaluate $\int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$ by using Simpson's 1/3 rule, taking $\Delta x = \Delta y = 0.25$. (16)

14. (a) i Using Taylor series method, find y at $x = 0.1$, given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$, **correct** to **4** decimal places. (8)

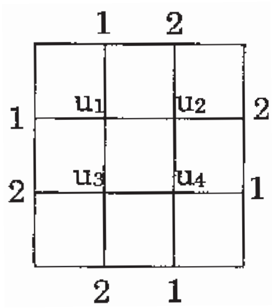
(ii) Using Runge–Kutta method of **fourth** order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ **given** $y(0) = 1$ at $x = 0.2$. (8)

Or

(b) Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, find $y(0.3)$ by Runge–Kutta method of **order** four **and** $y(0.4)$ using Milne's predictor–corrector method. (16)

15. (a) (i) Solve the equation $y'' = x + y$ **with** boundary conditions $y(0) = y(1) = 0$, numerically taking $\Delta x = 0.25$. (8)

(ii) Solve $u_{xx} + u_{yy} = 0$ numerically for the following mesh with boundary conditions as shown below : (8)



Or

(b) (i) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ **over the square** mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $a = 0$ on the boundary and mesh length of 1 unit. (10)

(ii) Solve $\frac{}{\partial x^2} - 2\frac{\partial u}{\partial t}$ **given**

$$u(0, t) = 0,\ u(4, t) = 0,\ u(x, 0) = x(4 - x),$$

taking $\Delta x = 1$ and $\Delta t = 1$. Find the value of u upto $t = 3$ using Bender–Schmidt's explicit **finite difference** scheme. (6)