**C 3275**

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fourth Semester

(Regulation 2004)

Aeronautical Engineering

MA 1251 — NUMERICAL METHODS

(Common to Civil Engineering/Mechatronics Engineering/Metallurgical Engineering/Electrical and Electronics Engineering/Petroleum Engineering)

(Common to B.E. (Part-Time) - Third Semester - Regulation 2005))

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the criterion for the convergence of Newton-Raphson method?

2. Write down the condition for the convergence of Gauss-Seidel iteration scheme.

3. If $f(x)=\frac{1}{x^2}$, find $f(a,b)$ and $f(a,b,c)$ by using divided differences.

4. Using Lagrange's interpolation, find the polynomial through (0, 0), (1, 1) and (2, 2).

5. State the formula of Simpson's $\frac{3}{8}$th rule.

6. Write Newton's forward difference formula to find the derivatives $\left(\frac{dy}{dx}\right)_{x=x_0}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$

7. Write Runge-Kutta's 4th order formula to solve $\frac{dy}{dx}=f(x,y)$ with $y(x_0)=y_0$.

8. Write Taylor's series formula to solve $y' = f(x, y)$ with $y(x_0) = y_0$.

9. Write down one dimensional wave equation and its boundary conditions.

10. State the explicit formula for the one dimensional wave equation with $1 - \lambda^2 a^2 = 0$ where $\lambda = \frac{k}{h}$ and $a^2 = T/m$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the positive root of $2x^3 - 3x - 6 = 0$ that lies between 1 and 2 by using Newton-Raphson method. (8)

(ii) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$ by Gauss-Jordan method. (8)

Or

(b) (i) By using Gauss-Seidel method, solve the following system of equations $6x + 3y + 12z = 35$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$. (8)

(ii) Find, by power method, the largest eigen value and the eigen vector of the matrix $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ (8)

12. (a) (i) Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data : (8)

| | | | | |
|---|---|---|---|---|
| $x$ : | 1 | 2 | 7 | 8 |
| $f(x)$ : | 1 | 5 | 5 | 4 |

(ii) From the following table, find the value of tan 45° 15' by Newton's forward interpolation formula. (8)

| | | | | | | |
|---|---|---|---|---|---|---|
| $x^\circ$ : | 45 | 46 | 47 | 48 | 49 | 50 |
| $\tan x^\circ$ : | 1.00000 | 1.03553 | 1.07237 | 1.11061 | 1.15037 | 1.19175 |

Or

(b) Fit the cubic spline for the data : (16)

| | | | | |
|---|---|---|---|---|
| $x$ : | 0 | 1 | 2 | 3 |
| $f(x)$ : | 1 | 2 | 9 | 28 |

13. (a) (i) Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ with $h = k = 0.2$ by using trapezoidal rule. (8)

(ii) From the following table, find the value of $x$ for which $f(x)$ is maximum. Also find the maximum value. (8)

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 60 | 75 | 90 | 105 | 120 |
| $f(x)$ : | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

Or

(b) (i) Using Romberg's rule, evaluate $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places by taking $h = 0.5$, 0.25 and 0.125. (8)

(ii) By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \sin x dx$ by using Simpson's $\frac{1}{3}$rd rule. Is it possible to evaluate the same by Simpson's $\frac{3}{8}$th rule. Justify your answer. (8)

14. (a) (i) Using Taylor's series method, find $y$ when $x = 1.1$ and 1.2 from $\frac{dy}{dx} = xy^{1/3}$, $y(1) = 1$. (4 decimal places). (8)

(ii) By using Adam's pc method find $y$ when $x = 0.4$, given $\frac{dy}{dx} = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$. (8)

Or

(b) (i) Using Runge-Kutta method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, given $y(0) = 1$ at $x = 0.2$. Take $h$ = 0.2. (8)

(ii) Find the value of $y$ when $x$ = 0.1 and 0.2, given $\frac{dy}{dx} = x^2 + y^2$ with $y = 1$ when $x = 0$. Use modified Euler's method. (8)

15. (a) Solve the Laplace's equation over the square mesh of side 4 units, satisfying the boundary conditions : (16)

$u(0, y) = 0,\ 0 \le y \le 4;\ u(4, y) = 12 + y,\ 0 \le y \le 4;$

$u(x, 0) = 3x,\ 0 \le x \le 4;\ u(x, 4) = x^2,\ 0 \le x \le 4.$

Or

(b) (i) Derive Bender-Schmidt for solving $u_{xx} - au_t = 0$ with the b.cs. $u(0, t) = T_0$; $u(l, t) = T_l$ and $u(x, 0) = f(x)$ for $0 < x < l$. Also find corresponding recurrence equation. (8)

(ii) By finite difference method, solve $\frac{d^2y}{dx^2} + x^2 y = 0$ with the b.cs $y(0) = 0$ and $y(1) = 1$. $h = 0.25$. (8)