UNIT-I CURVE FITTING AND THEORY OF EQUATIONS

Part-A

1. Define linear law.

Solution: The relation between the variables x & y is liner.

Let $y = ax + b$ ... (1)

If the points (x_i, y_i) are plotted in the graph sheet, they should lie on a straight line. 'a' is the slope of a straight line $a = \frac{y_2 - y_1}{x_2 - x_1}$, and 'b' can be found by knowing the y-intercept or by substituting any point on the line. Substituting a & b in equation (1), we get the straight line required.

2. Reduce the equation $\boldsymbol{y = ax^n}$ into linear form.

Solution: Given $y = ax^n$ ... (1)

Taking logarithm on both sides, we get

$$\log_{10} y = \log_{10} ax^n$$

$$\log_{10} y = \log_{10} a + \log_{10} x^n$$

$$\log_{10} y = \log_{10} a + n \log_{10} x$$

$$Y = A + B\,X \; is\ the\ required\ linear\ form.$$

$where\ Y = \log_{10} y, A = \log_{10} a, B = n, and = \log_{10} x$

3. Reduce the equation $\boldsymbol{y = a\,e^{bx}}$ into linear form.

Solution: Given $y = a\,e^{bx}$ ... (1)

Taking logarithm on both sides, we get

$$\log_{10} y = \log_{10} ae^{bx}$$

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

$$Y = A + Bx \; is\ the\ required\ linear\ form.$$

$where\ Y = \log_{10} y, A = \log_{10} a, and\ B = b \log_{10} e$

4. **Reduce the equation $\boldsymbol{y = a + bx^n}$ into linear form.**

Solution: Given $y = a + bx^n$ ... (1)

$$y = a + bX \; is\ the\ required\ linear\ form.$$

where $X = x^n$

5. what are the points in getting an emperical equation?

Solution:

i) The determination of the form of the equation.

ii) The evaluation of the constants in the equation.

6. Explain the method of group of averages while fitting a straight line $y = a + bx$.

Solution:

i) Divide the given data in to two groups so that both contain the same number of points.

ii) Find the average values $\bar{x}$ and $\bar{y}$ for the two groups

iii) Substituting the average values in $\bar{y} = a + b\bar{x}$

iv) Solving the equations got from step (iii) we get the values of a and b.

v) Substituting the values of a and b in $y = a + bx$ we get the required solution.

7. **What are the drawbacks of method of group of averages?**

Solution:

The drawbacks of method of group of averages were

i) The answer is not unique since the grouping can be done in many ways.

ii) The equation got is only an approximate fit to the data given.

8. Reduce the equation $y = a + bx + cx^2$ into linear form.

Solution:

Assume that a particular point (x_1, y_1) satisfies equation $y = a + bx + cx^2$ ... (1)

$$y_1 = a + bx_1 + c\,x_1^2 \quad \ldots (2)$$

$$(1) - (2) \Longrightarrow \; y - y_1 = b(x - x_1) + c(x^2 - x_1^2)$$

$$\frac{y - y_1}{x - x_1} = b + c(x + x_1)$$

$$Y = b + c\,X \; is\ the\ required\ linear\ form.$$

Where $Y = \frac{y - y_1}{x - x_1}$ and $X = x + x_1$.

9. Reduce the equation $y = ax^b + c$ into linear form.

Solution: The linear form is $Y = A + bX$

$$where \quad A = \log_{10} a \;\;, \;\; X = \log_{10} x\,, Y = \log_{10}(y - c)\,, \;\; c = \frac{y_1 y_3 - y_2^2}{y_1 + y_3 - 2y_2}$$

Here y_1, y_2 & y_3 are the corresponding values of three consecutive points x_1, x_2 & x_3 which are in Geometric progression $(x_2^2 = x_1 x_3)$.

10. Reduce the equation $y = ab^x + c$ into linear form.

Solution: The linear form is $Y = A + Bx$

$$where \quad A = \log_{10} a \;\;, \;\; B = \log_{10} b\,, \;\; Y = \log_{10}(y - c)\,, \;\; c = \frac{y_1 y_3 - y_2^2}{y_1 + y_3 - 2y_2}$$

Here y_1, y_2 & y_3 are the corresponding values of three consecutive points x_1, x_2 & x_3 which are in Arithmetic progression $(2x_2 = x_1 + x_3)$.

UNIT-I CURVE FITTING AND THEORY OF EQUATIONS

11. Reduce the equation $y = ae^{bx} + c$ into linear form.

Solution: The linear form is $Y = A + Bx$

$$where \quad A = \log_{10} a \ , \ B = b \ \log_{10} e \, , \ Y = \log_{10}(y - c) \, , \ c = \frac{y_1 y_3 - y_2^2}{y_1 + y_3 - 2y_2}$$

Here y_1, y_2 & y_3 are the corresponding values of three consecutive points x_1, x_2 & x_3 which are in Arithmetic progression $(2x_2 = x_1 + x_3)$.

12. Explain the method principle of least squares while fitting a straight line $y = ax + b$.

Solution :

i) Form the normal equations:

The normal equations for $y = ax + b$ are

$$a \sum x + n\, b = \sum y$$

$$a \sum x^2 + \ b \sum x = \sum x\, y$$

Where n is the number of observations.

ii) Find the values of $\sum x$, $\sum x^2$, $\sum y$ & $\sum xy$ by forming the table from the given data & substituting these values in the normal equations.

iii) Solving these equations, we get the values of a & b .

iv) Substituting the values of a & b in $y = ax + b$ we get the required solution.

13. Explain the method principle of least squares while fitting a second degree curve or parabola $y = ax^2 + bx + c$

Solution :

i) Form the normal equations:

The normal equations for $y = ax + b$ are

$$a \sum x^2 + \ b \sum x + nc = \sum y$$

$$a \sum x^3 + \ b \sum x^2 + c \sum x = \sum xy$$

$$a \sum x^4 + \ b \sum x^3 + c \sum x^2 = \sum x^2 y$$

Where n is the number of observations.

ii) Find the values of $\sum x$, $\sum x^2$, $\sum x^3$, $\sum x^4$, $\sum y$, $\sum xy$ & $\sum x^2 y$ by forming the table from the given data & substituting these values in the normal equations.

iii) Solving these equations, we get the values of a, b & c .

iv) Substituting the values of a & b in $y = ax^2 + bx + c$ we get the required solution.

14. Explain method of moments while fitting a straight line $y = ax + b$.

Solution:

i) Form the observation equations:

The observation equations for $y = ax + b$ are

$$\frac{a}{2}(\beta^2 - \alpha^2) + b(\beta - \alpha) = \Delta x \sum y$$

$$\frac{a}{3}(\beta^3 - \alpha^3) + \frac{b}{2}(\beta^2 - \alpha^2) = \Delta x \sum xy$$

where $\alpha = x_1 - \frac{1}{2}\Delta x, \beta = x_n + \frac{1}{2}\Delta x \; and \; \Delta x = x_i - x_{i-1}, i = 1,2,3, \dots, n$.

ii) Find the values of $\sum y$ & $\sum xy$ by forming the table from the given data & substituting these values in the observation equations.

iii) Solving these equations, we get the values of a & b .

iv) Substituting the values of a & b in $y = ax + b$ we get the required solution.

15. Explain the method of moments while fitting a second degree curve or parabola $y = ax^2 + bx + c$

Solution:

i) Form the observation equations:

The observation equations for $y = ax^2 + bx + c$ are

$$\frac{a}{3}(\beta^3 - \alpha^3) + \frac{b}{2}(\beta^2 - \alpha^2) + c(\beta - \alpha) = \Delta x \sum y$$

$$\frac{a}{4}(\beta^4 - \alpha^4) + \frac{b}{3}(\beta^3 - \alpha^3) + \frac{c}{2}(\beta^2 - \alpha^2) = \Delta x \sum xy$$

$$\frac{a}{5}(\beta^5 - \alpha^5) + \frac{b}{4}(\beta^4 - \alpha^4) + \frac{c}{3}(\beta^3 - \alpha^3) = \Delta x \sum x^2 y$$

where $\alpha = x_1 - \frac{1}{2}\Delta x, \beta = x_n + \frac{1}{2}\Delta x \; and \; \Delta x = x_i - x_{i-1}, i = 1,2,3, \dots, n$.

ii) Find the values of $\sum y$, $\sum xy$ & $\sum x^2 y$ by forming the table from the given data & substituting these values in the observation equations.

iii) Solving these equations, we get the values of a , b and c .

iv) Substituting the values of a , b and c in $y = ax^2 + bx + c$ we get the required solution.

16. Define a polynomial equation or algebraic equation.

Solution:

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n = 0$ is called an algebraic equation or polynomial equation of the nth degree, if $a_0 \neq 0$.

17. Solve $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, given $-1 + i$ is a root.

Solution: Since $-1 + i$ is a root, $-1 - i$ is also a root.

$$[x - (-1 + i)][x - (-1 - i)] = (x + 1)^2 + 1 = x^2 + 2x + 2$$

when the polynomial is divided by $x^2 + 2x + 2$, the remainder is zero.

$$x^4 + 4x^3 + 5x^2 + 2x - 2 = (x^2 + 2x + 2)(x^2 + ax - 1)$$

Equating the coefficient of x^3 on both sides, we get $a + 2 = 4 \Rightarrow a = 2$

$$\therefore f(x) = (x^2 + 2x + 2)(x^2 + 2x - 1)$$

Solving $x^2 + 2x - 1 = 0$, we get $x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

The roots are $-1 \pm i, -1 \pm \sqrt{2}$.

18. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$, given $2 + \sqrt{3}$ is a root.

Solution: Since $2 + \sqrt{3}$ is a root, $2 - \sqrt{3}$ is also a root.

$$[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})] = (x - 2)^2 - 3 = x^2 - 4x + 1$$

when the polynomial is divided by $x^2 - 4x + 1$, the remainder is zero.

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = (x^2 - 4x + 1)(x^2 + ax + 1)$$

Equating the coefficient of x^3 on both sides, we get $a - 4 = -10 \Rightarrow a = -6$

$$\therefore f(x) = (x^2 - 4x + 1)(x^2 - 6x + 1)$$

Solving $x^2 - 6x + 1 = 0$, we get $x = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$

The roots are $2 \pm \sqrt{3}, 3 \pm 2\sqrt{2}$.

19. Write down the relations between the roots and the coefficients of equation?

Solution:

Let the roots of $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$ be $\alpha_1, \alpha_2, \dots, \alpha_n$. then

$$\sum \alpha_1 = -\frac{a_1}{a_0}; \sum \alpha_1\alpha_2 = \frac{a_2}{a_0}; \sum \alpha_1\alpha_2\,\alpha_3 = -\frac{a_3}{a_0}; \dots ; \alpha_1\alpha_2 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

20. Find the condition that the roots of the equation $x^3 + px^2 + qx + r = 0$ may be in arithmetical progression.

Solution:

Let the roots in arithmetic progression be $\alpha - d, \alpha, \alpha + d$.

$$Sum\ of\ the\ roots\ = \alpha - d + \alpha + \alpha + d = -\frac{p}{1} = -p \Rightarrow 3\alpha = -p \Rightarrow \alpha = -\frac{p}{3}$$

Now $x = \alpha$ is a root of the given equation.

$$\alpha = -\frac{p}{3}\ should\ satisfy\ the\ equation$$

$$\left(-\frac{p}{3}\right)^3 + p\left(-\frac{p}{3}\right)^2 + q\left(-\frac{p}{3}\right) + r = 0$$

$$-\frac{p^3}{27} + \frac{p^3}{9} - \frac{pq}{3} + r = 0$$

$$2p^3 - 9pq + 27r = 0$$

21. Find the condition that the roots of the equation $x^3 + px^2 + qx + r = 0$ may be in geometrical progression.

Solution:

Let the roots in geometric progression be $\frac{\alpha}{d}, \alpha, \alpha d$.

$$Product\ of\ the\ roots\ = \frac{\alpha}{d}.\alpha.\alpha d = -\frac{r}{1} = -r \Rightarrow \alpha^3 = -r \dots (1)$$

$x = \alpha$ is a root of $f(x) = 0 \Rightarrow \alpha^3 + p\alpha^2 + q\alpha + r = 0$

$$-r + p\alpha^2 + q\alpha + r = 0\ (from\ (1))$$

$$p\alpha^2 + q\alpha = 0 \Rightarrow \alpha(p\alpha + q) = 0$$

$$(p\alpha + q) = 0\ since\ \alpha \neq 0$$

$$\therefore \alpha = -\frac{q}{p}$$

Putting it in (1), we get

$$-\frac{q^3}{p^3} = -r \Rightarrow p^3 r = q^3$$

22. Find the condition that the roots of the equation $x^3 + px^2 + qx + r = 0$ may be in harmonic progression.

Solution:

Let the roots of the equation (1) are in arithmetic progression be $\alpha - d, \alpha, \alpha + d$.

$$Sum\ of\ the\ roots\ = \alpha - d + \alpha + \alpha + d = -\frac{q}{r} = -p \Rightarrow 3\alpha = -\frac{q}{r} \Rightarrow \alpha = -\frac{q}{3r}$$

Now $x = \alpha$ is a root of the given equation.

$$\alpha = -\frac{p}{3}\ should\ satisfy\ the\ equation$$

$$r\left(-\frac{q}{3r}\right)^3 + q\left(-\frac{q}{3r}\right)^2 + p\left(-\frac{q}{3r}\right) + 1 = 0$$

$$-\frac{q^3}{27r^2} + \frac{q^3}{9r^2} - \frac{pq}{3r} + 1 = 0$$

$$2q^3 - 9pqr + 27r^2 = 0$$

23. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ find the condition if $\alpha + \beta = 0$.

Solution: Given $\alpha + \beta = 0$, we know that $\alpha + \beta + \gamma = -p \Rightarrow 0 + \gamma = -p \Rightarrow \gamma = -p$

γ satisfies the given equation, therefore $-p^3 + p(-p)^2 + q(-p) + r = 0 \Rightarrow -p^3 + p^3 - pq + r = 0$

$$r = pq$$

24. Form the third degree equation, two of whose roots are $1 - i$ and 2.

Solution: Since $1 - i$ is a root $1 + i$ is also a root. Therefore, the equation of degree three is

$$[x - (1 - i)][x - (1 + i)][x - 2] = 0$$

$$(x^2 - 2x + 2)(x - 2) = 0 \Rightarrow x^3 - 4x^2 + 6x - 4 = 0$$

25. If a, b, c are the roots of $x^3 + px^2 + qx + r = 0$, find the equation whose roots are ab, bc, ca.

Solution:

We know that

$$\sum a = -p, \sum ab = q, abc = -r$$

Let $y = ab = \frac{abc}{c} = -\frac{r}{c} = -\frac{r}{x}$

$$\therefore x = -\frac{r}{y} \text{ satisfies the given equation}$$

$$\left(-\frac{r}{y}\right)^3 + p\left(-\frac{r}{y}\right)^2 + q\left(-\frac{r}{y}\right) + r = 0$$

$$y^3 - qy^2 + pry - r^2 = 0 \text{ is the required equation}$$

26. If a, b, c are the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\sum a^2 b$.

Solution: We know that, $\sum a = -p, \sum ab = q, abc = -r$

$$\sum a^2 b = \left(\sum ab\right)\left(\sum a\right) - 3abc = q(-p) - 3(-r) = 3r - pq$$

27. If a, b, c are the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\sum a^2$.

Solution: We know that, $\sum a = -p, \sum ab = q, abc = -r$

$$\sum a^2 = \left(\sum a\right)^2 - 2\left(\sum ab\right) = p^2 - 2q$$

28. If a, b, c are the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\sum a^3$.

Solution: We know that, $\sum a = -p, \sum ab = q, abc = -r$

$$\sum a^3 = \left(\sum a\right)^3 - 3\left(\sum ab\right)\left(\sum a\right) + 3abc = -p^3 - 3q(-p) + 3(-r) = -p^3 + 3pq - 3r$$

29. If α, β, γ are the roots of $x^3 - 3ax + b = 0$, show that $\sum(\alpha - \beta)(\alpha - \gamma) = 9a$.

Solution: We have $\sum \alpha = 0, \sum \alpha\beta = -3a, \alpha\beta\gamma = -b$

$$\sum(\alpha - \beta)(\alpha - \gamma) = \sum(\alpha^2 - \alpha\beta - \alpha\gamma + \beta\gamma)$$

$$= \sum \alpha^2 - \sum \alpha\beta - \sum \alpha\gamma + \sum \beta\gamma$$

$$= \left(\sum \alpha\right)^2 - 2\sum \alpha\beta - \sum \alpha\beta - \sum \alpha\gamma + \sum \beta\gamma$$

$$= 0 - 2(-3a) - (-3a) - (-3a) - 3a = 9a$$

30. Define reciprocal equation.

Solution:

If an equation $f(x) = 0$ remains unaltered when x is changed to $\frac{1}{x}$, then it is called a reciprocal equation.

Part-B

1. Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

2. Solve $x^5 + 4x^4 + x^3 + x^2 + 4x + 1 = 0$.

3. Solve $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

4. Solve $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

5. Solve $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$.

6. Solve $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$.

7. Solve $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.

8. Solve $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$.

9. Solve $x^3 - 19x^2 + 114x - 216 = 0$, given that the roots are in geometric progression.

10. Solve $2x^3 - x^2 - 22x - 24 = 0$ given that two of its roots are in the ratio 3 : 4.

11. Solve $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ whose roots are in arithmetic progression.

12. Solve $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that two of its roots are equal in magnitude but opposite in sign.

13. Find the condition that the equation $x^4 + px^3 + qx^2 + rx + s = 0$

(i) may have a pair of roots whose sum is zero.

(ii) be such that the sum of two roots equals the sum of the other two roots.

(iii) may have roots such that the product of two roots equals the product of the other two roots.

14. Solve $6x^3 - 11x^2 + 6x - 1 = 0$ given that the roots are in harmonic progression.

15. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ form the equation whose roots are

$(i)\ \alpha + \beta, \beta + \gamma, \gamma + \alpha.\quad (ii)\ \frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}.\quad (iii)\ \frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}.$

16. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of the equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$, then form the equation whose roots are

$(i) - \alpha_1, -\alpha_2, \ldots, -\alpha_n \quad (ii)\ k\alpha_1, k\alpha_2, \ldots, k\alpha_n \quad (iii)\ \frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \ldots, \frac{1}{\alpha_n}$

UNIT-I CURVE FITTING AND THEORY OF EQUATIONS

17. Fit a straight line and a parabola to the following data by the method of least squares

x:	0	1	2	3	4
y:	1	1.8	1.3	2.5	6.3

18. Fit a straight line $y = ax + b$ to the following data by the method of group averages

x:	0	5	10	15	20	25
y:	12	15	17	22	24	30

19. Fit a curve of the form $y = ax^n$ from the following data by method of group averages

x:	1.68	2.45	3.08	4.09	4.97	5.95	7.39	9.00
y:	0.013	0.027	0.042	0.073	0.108	0.151	0.233	0.341

20. From the following table fit a curve of the form $y = ab^x$ by method of group averages

x:	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
y:	2.9	4.0	5.7	8.9	12.4

21. Find the best values of a and b if the following data are related by $y = ax + bx^2$ by method of group averages

x:	1	2	3	4	5	6
y:	2.6	5.4	8.7	12.1	16	20.2

22. Fit a curve of the form $y = ab^x$ to the data by method of least squares

x:	1	2	3	4	5	6
y:	151	100	61	50	20	8

23. Fit the curve of the form $y = ae^{bx}$ to the following data by method of least squares

x:	1	2	3	4
y:	1.65	2.70	4.50	7.35

24. Fit a straight line and parabola by the method of moments to the data

x:	1	2	3	4
y:	16	19	23	26

25. Fit a straight line and parabola by the method of moments to the data

x:	1	3	5	7	9
y:	1.5	2.8	4.0	4.7	6.0

26. Fit a curve of the form $v = at^2 + bt + c$ given the following data by method of group averages

t:	1	2	3	4	5	6
v:	3	4	7	12	21	32

UNIT-I CURVE FITTING AND THEORY OF EQUATIONS