NUMERICAL METHODS AND LINEAR PROGRAMMING

UNIT-III

TWO MARKS AND ASSIGNMENT QUESTIONS

1. **Define finite differences**.

Solution :

The first differences of y are denoted by Δy.

That is $\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_2$

$\Delta y_2 = y_3 - y_1, \dots ..$

...

$\Delta y_{n-1} = y_n - y_{n-1}$

$Higher\ differences$:

$\Delta^2 y_0 = \Delta\, \Delta(y_0) = \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta\, \Delta(y_1) = \Delta(y_2 - y_1) = \Delta y_2 - \Delta y_1$

$\Delta^2 y_{n-1} = \Delta\, \Delta(y_{n-1}) = \Delta(y_n - y_{n-1}) = \Delta y_2 - \Delta y_1$

2. Define forward difference operator Δ.

Solution:

Consider the arguments $x_0, x_0 + h,\ x_0 + 2h, \dots \dots ., x_0 + nh$

$$x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = \cdots = h$$

h is called interval of differencing.

The forward difference operator Δ is defined as

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta^2 f(x) = \Delta[\Delta f(x)] = \Delta\,[f(x+h) - f(x)]$$

$$= \Delta f(x+h) - \Delta f(x) = [f(x+2h) - f(x-h)] - [f(x+h) - f(x)]$$

$$= f(x+2h) - 2f(x-h) + f(x)$$

And so on.

3. Define backward difference operator ∇.

Solution:

The forward difference operator ∇ is defined as

$$\nabla f(x) = f(x) - f(x+h)$$

$$\nabla^2 f(x) = \nabla[\nabla f(x)] = \nabla\,[f(x) - f(x-h)]$$

$$= \nabla f(x) - \nabla f(x-h) = [f(x) - f(x-h)] - [f(x-h) - f(x-2h)]$$

$$= f(x) - 2f(x-h) + f(x-2h)$$

4. Form the forward difference table.

Solution:

The finite forward differences of a function are represented bythe table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
x_0	y_0						
		Δy_0					
x_1	y_1		$\Delta^2 y_0$				
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$	
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$		$\Delta^6 y_0$
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$	
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		
		Δy_4		$\Delta^3 y_3$			
x_5	y_5		$\Delta^2 y_4$				
		Δy_5					
x_6	y_6						

5. Form the backward difference table.

Solution:

The finite backward differences of a function are represented bythe table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
x_0	y_0						
		∇y_1					
x_1	y_1		$\nabla^2 y_2$				
		∇y_2		$\nabla^3 y_3$			
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$		
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$	
x_3	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$		$\nabla^6 y_6$
		∇y_4		$\nabla^3 y_5$		$\nabla^5 y_6$	
x_4	y_4		$\nabla^2 y_5$		$\nabla^4 y_6$		
		∇y_5		$\nabla^3 y_6$			
x_5	y_5		$\nabla^2 y_6$				
		∇y_6					
x_6	y_6						

6. Define Central differnce operator δ.

Solution:

The central difference operator δ is defined as

$$\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$$

$$or \quad \delta y_x = y_{x+\frac{h}{2}} - y_{x-\frac{h}{2}}$$

7. Define Shiffting or Displacement or Translation operator E.

Solution :

The Shiffting or Displacement or Translation operator E is defined by

$$Ef(x) = f(x+h) \quad or \quad Ey_x = y_{x+h}$$

Also $Ey_1 = y_2, \ Ey_2 = y_3, \dots \dots$

$$E^2 y_x = E(Ey_x) = E(y_x + h\,) = y_{x+h+h} = y_{x+2h}$$

$$lll^y \quad E^n y_x = y_{x+nh} \quad or \quad E^n f(x) = f(x+nh)$$

8. Define Inverse operator E^{-1}.

Solution :

The Inverse operator E^{-1} is defined by

$$E^{-1} f(x) = f(x-h) \quad or \quad E^{-1} y_x = y_{x-h}$$

$$E^{-r} y_x = y_{x-rh} \quad or \quad E^{-n} f(x) = f(x-nh)$$

9. What are the properties of operators.

Solution :

(a). The operators $\Delta, \nabla, \delta, E$ are all linear operators.

Proof: $\Delta[af(x) + bg(x)] = \{a[f(x+h) + bg(x+h)] - b[f(x) + g(x)]\}$

$$= \{a[f(x+h) - f(x)] + b[g(x+h) - g(x)]\}$$

$$\Delta[af(x) + bg(x)] = a\,\Delta f(x) + b\Delta g(x)$$

Hence Δ is linear operator.

Case : 1 Put $a = b = 1$, we get $\Delta[f(x) + g(x)] = \Delta f(x) + \Delta g(x)$

Case : 1 Put $b = 0$, we get $\Delta[af(x)] = a\,\Delta f(x)$

(b). The operator is distributive over addition.

$\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x) = \Delta^n \Delta^m f(x)$.

10. Find the relation between the operators Δ $\boldsymbol{and}$ $\boldsymbol{E}$.

Solution :

We know that $\Delta f(x) = f(x+h) - f(x)$

$$= Ef(x) - f(x)$$

$$\Delta f(x) = f(x)[E-1]$$

$$\Delta = \boldsymbol{E - 1} \quad \boldsymbol{or} \quad \boldsymbol{E = 1 + \Delta}$$

11. Find the relation between the operators ∇ $\boldsymbol{and}$ $\boldsymbol{E}$.

Solution :

We know that $\nabla f(x) = f(x) - f(x-h)$

$$= f(x) - E^{-1} f(x)$$

$$\nabla f(x) = f(x)[1 - E^{-1}]$$

$$\boldsymbol{\nabla = 1 - E^{-1}} \quad \boldsymbol{or} \quad \boldsymbol{E^{-1} = 1 - \nabla} \quad \textbf{or} \quad \mathbf{E = (1 - \nabla)^{-1}}$$

12. Find the relation between the operators $\boldsymbol{E}$ $\boldsymbol{and}$ $\boldsymbol{\delta}$.

Solution :

We know that $\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$

$$= E^{\frac{1}{2}} f(x) - E^{-\frac{1}{2}} f(x)$$

$$\delta f(x) = f(x)\left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right]$$

$$\boldsymbol{\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}}$$

$$Also \quad \boldsymbol{\delta} = \frac{E^{\frac{1}{2}}}{E^{\frac{1}{2}}}\left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right] = E^{\frac{1}{2}}\left[\frac{E^{\frac{1}{2}}}{E^{\frac{1}{2}}} - \frac{E^{\frac{-1}{2}}}{E^{\frac{1}{2}}}\right]$$

$$= E^{\frac{1}{2}}\left[1 - E^{-1}\right] = E^{\frac{1}{2}}\,\nabla$$

$$\boldsymbol{\delta} = E^{\frac{1}{2}}\;\nabla$$

$$And \quad \boldsymbol{\delta} = \frac{E^{\frac{-1}{2}}}{E^{\frac{-1}{2}}}\left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right] = E^{\frac{-1}{2}}\left[\frac{E^{\frac{1}{2}}}{E^{\frac{-1}{2}}} - \frac{E^{\frac{-1}{2}}}{E^{\frac{-1}{2}}}\right]$$

$$= E^{\frac{1}{2}}\left[E - 1\right] = E^{\frac{-1}{2}}\;\Delta$$

$$\boldsymbol{\delta} = E^{\frac{-1}{2}}\;\Delta$$

13. Find the general term of the sequence $2, 9, 28, 65, 126, 217$also find the 7^{th} term.

Solution :

To find general :

W.k.t $y_n = y_0 + nC_1\;\Delta y_0 + nC_2\;\Delta^2 y_0 + nC_3\;\Delta^3 y_0 + nC_4\;\Delta^4 y_0 + \cdots \ldots + \;\Delta^n y_0$

$$= y_0 + n\;\Delta y_0 + \frac{n(n-1)}{2!}\;\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\;\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\;\Delta^4 y_0 + \cdots \ldots + \;\Delta^n y_0$$

$$= 2 + n\;(7) + \frac{n(n-1)}{2}\;12 + \frac{n(n-1)(n-2)}{3!}\;6 + \frac{n(n-1)(n-2)(n-3)}{4!}\;0 + \cdots \ldots$$

$$= 2 + n\;(7) + (n^2 - n)\,6 + (n^2 - n)(n-2)$$

$$= 2 + 7n + 6n^2 - 6n + n^3 - 3n^2 + 2n$$

$$y_n = n^3 + 3n^2 + 3n + 2$$

To find 7^{th} term : Put $n = 6$.

$$y_6 = 6^3 + 3(6)^2 + 3(6) + 2$$

$$= 344$$

14. Prove that $E\nabla = \Delta = \nabla E$.

Proff:

$$(E\nabla)f(x) = E\big(\nabla f(x)\big) = E[f(x) - f(x-h)]$$

$$= Ef(x) - Ef(x-h)$$

$$= f(x+h) - f(x-h+h)$$

$$= f(x+h) - f(x)$$

$$E\nabla f(x) = \Delta f(x)$$

Hence $E\nabla = \Delta$.

Also $\nabla E\, f(x) = \nabla\, f(x+h) = f(x+h) - f(x+h-h)$

$$= f(x+h) - f(x) = \Delta f(x)$$

$$\nabla E f(x) = \Delta f(x)$$

Hence $\nabla E = \Delta$. There fore $E\nabla = \nabla E = \Delta$.

15. Prove that $\boldsymbol{\delta E^{\frac{1}{2}}} = \Delta$**.**

Proff:

$$\left(\delta E^{\frac{1}{2}}\right) y_x = \delta\left(E^{\frac{1}{2}} y_x\right) = \delta\left(y_{x+\frac{h}{2}}\right)$$

$$= \left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right]\left(y_{x+\frac{h}{2}}\right)$$

$$= \left(E^{\frac{1}{2}} y_{x+\frac{h}{2}} - E^{\frac{-1}{2}} y_{x+\frac{h}{2}}\right)$$

$$= y_{x+\frac{h}{2}+\frac{h}{2}} - y_{x+\frac{h}{2}-\frac{h}{2}}$$

$$= y_{x+h} - y_x$$

$$\left(\delta E^{\frac{1}{2}}\right) y_x = \Delta y_x$$

Hence $\delta E^{\frac{1}{2}} = \Delta$.

16. Prove that $\boldsymbol{\nabla}\Delta = \Delta - \boldsymbol{\nabla} = \boldsymbol{\delta^2}$**.**

Proff:

$$\nabla\Delta = (1 - E^{-1})(E - 1) = E - E^{-1}E - 1 + E^{-1}$$

$$= E - 1 - 1 + E^{-1} = E + E^{-1} - 2$$

$$= \left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right]^2$$

$$\boldsymbol{\nabla}\Delta = \boldsymbol{\delta^2}$$

$$\Delta - \nabla = (\mathrm{E} - 1) - (1 - \mathrm{E}^{-1})$$

$$= E + E^{-1} - 2$$

$$= \left[E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right]^2$$

$$\Delta - \boldsymbol{\nabla} = \boldsymbol{\delta^2}$$

17. Prove that $\frac{1}{2}\,\boldsymbol{\delta^2} + \boldsymbol{\delta}\sqrt{\mathbf{1} + \frac{\boldsymbol{\delta^2}}{\mathbf{4}}} = \Delta$**.**

Proff:

$$\frac{1}{2}\,\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}} = \frac{\delta}{2}\left[\delta + 2\sqrt{\frac{4 + \delta^2}{4}}\right]$$

$$= \frac{\delta}{2}\left[\delta + \frac{2}{2}\sqrt{4 + \delta^2}\right]$$

$$= \frac{\delta}{2}\left[\delta + \sqrt{4 + \left(E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right)^2}\right]$$

$$= \frac{\delta}{2}\left[\delta + \sqrt{4 + (E + E^{-1} - 2E^0)}\right]$$

$$= \frac{\delta}{2}\left[\delta + \sqrt{E + E^{-1} + 2}\right]$$

$$= \frac{\delta}{2}\left[\delta + \sqrt{\left(E^{\frac{1}{2}} + E^{\frac{-1}{2}}\right)^2}\right]$$

$$= \frac{\delta}{2}\left[\left(E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right) + \left(E^{\frac{1}{2}} + E^{\frac{-1}{2}}\right)\right]$$

$$= \frac{\delta}{2}\left[2\, E^{\frac{1}{2}}\right] = \left(E^{\frac{1}{2}} - E^{\frac{-1}{2}}\right)\left[E^{\frac{1}{2}}\right]$$

$$= E^{\frac{1}{2}}\, E^{\frac{1}{2}} - E^{\frac{-1}{2}}\, E^{\frac{1}{2}} = E - E^0$$

$$\frac{1}{2}\,\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}} = E - 1 = \Delta$$

18. Prove that $(\mathbf{1} + \Delta)(\mathbf{1} - \boldsymbol{\nabla}) = \boldsymbol{I}$.

Proff:

$$(1 + \Delta)(1 - \nabla) = E * E^{-1} = I$$

19. State Gregory-Newton's forwadr interpolation formula (or) Mewton's forward difference formula.

Solution :

The Newton's forwad interpolation formula for $f(x)$ is defined by

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + ..$$

$$where\ \ u = \frac{x - x_0}{h}$$

This formula is used to find the values of y nearer to the begining value of the table.

20. State Gregory-Newton's backward interpolation formula (or) Mewton's backward difference formula.

Solution :

The Newton's backward interpolation formula for $f(x)$ is defined by

$$y(x) = y_n + \frac{V}{1!}\nabla y_n + \frac{V(V+1)}{2!}\nabla^2 y_n + \frac{V(V+1)(V+2)}{3!}\nabla^3 y_n + \frac{V(V+1)(V+2)(V+3)}{4!}\nabla^4 y_n + ..$$

$$where\ \ V = \frac{x - x_n}{h}$$

This formula is used to find the values of y nearer to the ending value of the table.

21. State Gauss's forwad interpolation formula.

Solution :

The Gauss's forwad interpolation formula for $f(x)$ is defined by

$$y(x) = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_{-1} + \binom{u+1}{3}\Delta^3 y_{-1} + \binom{u+1}{4}\Delta^4 y_{-2} + ..$$

$$where\ \ u = \frac{x - x_0}{h}$$

This formula can be used only when u lies between $0 \quad and \quad 1.$

22. State Gauss's Backward interpolation formula.

Solution :

The Gauss's backwad interpolation formula for $f(x)$ is defined by

$$y(x) = y_0 + \binom{u}{1}\Delta y_{-1} + \binom{u+1}{2}\Delta^2 y_{-1} + \binom{u+1}{3}\Delta^3 y_{-2} + \binom{u+2}{4}\Delta^4 y_{-2} + ..$$

$$where\ \ u = \frac{x - x_0}{h}$$

This formula can be used only when u lies between $-1 \quad and \quad 0.$

23. State Bessel's formula.

Solution :

The bessel's formula for $f(x)$ is defined by

$$y(x) = y_0 + \left(\frac{y_0 + y_1}{2}\right) + \left(u - \frac{1}{2}\right)\Delta y_0 + \left[\frac{u(u-1))}{2!}\right]\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right)$$

$$+ \left[\frac{\left(u - \frac{1}{2}\right)u(u-1)}{3!}\right]\Delta^3 y_{-1} + \left[\frac{(u+1)u(u-1)(u-2)}{4!}\right]\left(\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}\right) + ..$$

$$where\ \ u = \frac{x - x_0}{h}$$

24. State Laplace-Everett formula.

Solution :

The Laplace-Everett formula for $f(x)$ is defined by

$$y(x) = \left[v\, y_0 + \frac{v(v^2 - 1^2)}{3!}\Delta^2 y_{-1} + \frac{v(v^2 - 1^2)(v^2 - 2^2)}{5!}\Delta^4 y_{-2} + \cdots\right] +$$

$$\left[u\, y_1 + \frac{u(u^2 - 1^2)}{3!}\Delta^2 y_0 + \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!}\Delta^4 y_{-1} + \cdots\right]$$

$$where\ \ u = \frac{x - x_0}{h} \quad and \quad v = 1 - u.$$

PART-B

1. Find the values of $y \ \ at \ \ x = 21 \ \ 7 \ \ x = 28$ from the following data.

X	20	23	26	29
Y	0.3420	0.3907	0.4384	0.4848

2. The population of a town is as follows.

Year	1941	1951	1961	1971	1981	1991
Population	20	24	29	36	46	51

Estimate the population increase during the period 1946 & 1976.

3. From the following table find $\theta \ \ at \ \ x = 43 \ \ \& \ \ x = 84.$

X	40	50	60	70	80	90
θ	184	204	226	250	276	304

4. Find a polynomial of degree four which takes the values

X	2	4	6	8	10
y	0	0	1	0	0

5. If $\sqrt{12500} = 111.803,\ \sqrt{12510} = 111.848,\ \sqrt{12520} = 111.892,\ \sqrt{12530} = 111.937,$

$find\ \sqrt{12516}$ by Gauss's backward & forward formula.

6. Using the following table, apply Gauss's forward and backward formula to get $f(3.75)$

X	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	24.145	22.043	20.225	18.644	17.262	16.047

7. The population of a town is as follows.

Year	1911	1921	1931	1941	1951
Population	15	20	27	39	52

Estimate the population increase during the period 1926.

8. Using Gauss forward formula to get y_{30} given that $y_{21} = 18.4708,\ y_{25} = 17.8144,$

$y_{29} = 17.1070,\ y_{33} = 16.3432,\ y_{37} = 15.5154.$

9. From the following table, estimate $e^{0.6444}$ correct to five decimal places using

(i). Bessel's Formula (ii). Laplace's formula.

X	0.61	0.62	0.63	0.64	0.65	0.66	0.67
e^x	1.840431	1.858928	1.877610	1.896481	1.915541	1.934792	1.954237

10. The following table gives the values of the probability integral $f(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\frac{x^2}{2}}dx$ for certain values of x. Find the value of this integral when $x = 0.5437$ using (i). Bessel's formula (ii) Evrett's formula.

X	0.51	0.52	0.53	0.54	0.55	0.56	0.57
$y = f(x)$	0.5292437	0.5378987	0.5464641	0.55549392	0.5633233	0.5716157	0.5798158

11. From the following table, find $f(34)$ using Everett's formula, Bessel's formula.

X	20	25	30	35	40
$f(x)$	11.4699	12.7834	13.7468	14.4982	15.0463

12. From the following table, find $\tan 16^0$,

X	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$f(x)$	0.0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

13. From the following table, estimate $f(337.5)$ by proper formula,

X	310	320	330	340	350	360
$f(x)$	2.4913617	2.5051500	2.5185139	2.5314789	2.5440680	2.5563025

14. Estimate the production of the year 1964 & 1966 fom the following table

$year$	1961	1962	1963	1964	1965	1966	1967
$Production$	200	220	260	–	350	–	430

15. Find the missing term in the following

X	1	2	3	4	5	6	7
y	2	4	8	–	32	64	128

16. Find y_6 if $y_0 = 9,\ y_1 = 18,\ y_2 = 20,\ y_3 = 24$ given that the third differences are constants.

17. First the first term of the series whoose second and subsequent terms are $8,\ 3,\ 0, -1,\ 0,$ .....

18. Find $f(x)$ from the table below. Also find $f(7)$.

19. Find the 7^{th} term of the sequence $2,\ 9,\ 28,\ 65,\ 126,\ 217$ and also find the general term.

20. Find the sixth term of the sequence $8,\ 12,\ 19,\ 29,\ 42, \ldots\ldots$