SRI RAMAKRISHNA INSTITUTE OF TECHNOLOGY, COIMBATORE- 10
DEPARTMENT OF SCIENCE AND HUMANITIES
SUBJECT: NUMERICAL METHODS & LINEAR PROGRAMMING (SEMESTER - IV)
IV- INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION
TWO MARKS & ASSIGNMENT

1. State Lagrange's interpolation formula.

Answer :

Let $y = f(x)$ be a function which takes the values

$y_0, y_1, \ldots \ldots . y_n$ $corresponding\ to\ x = x_0, x_1, x_2, \ldots \ldots \ldots$

Then , Lagrange's interpolation formula is

$$y(x) = \frac{(x - x_1)(x - x_2) \ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_n)}\ y_0 + \frac{(x - x_0)(x - x_2) \ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \ldots (x_1 - x_n)}\ y_1$$
$$+ \frac{(x - x_0)(x - x_1) \ldots (x - x_n)}{(x_2 - x_0)(x_2 - x_1) \ldots (x_2 - x_n)}\ y_2 + \cdots \ldots . + \frac{(x - x_0)(x - x_1) \ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1) \ldots (x_n - x_{n-1})}\ y_n$$

2. What is the Lagrange's interpolation formula to find y, if three sets of values $(x_0, y_0), (x_1, y_1)$ & (x_2, y_2) are given.

Answer :

$$y(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}\ y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}\ y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}\ y_2$$

3. What is the assumption we make when Lagrange's formula is used?

Answer :

Lagrange's interpolation formula can be used whether the vales of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. What advantages has Lagrange's interpolation formula over Newton?

Answer :

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced can also be used when the differences of the independent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not and whether the difference of y become smaller or not.

5. What is the disadvantage in practice in applying Lagrange's interpolation formula?

Answer :

Through Lagrange's interpolation formula is simple and easy to remember, its application is not speedy. It requires close attention to sign and there is always a chance of committing some error due to a number of positive and negative signs in the numerator and the denominator.

6. What is inverse interpolation?

Answer :

Suppose we are given a table of vales of x and y. Direct interpolation is the process of finding the values of y corresponding to a value of x, not present in the table. Inverse interpolation is the process of finding the values of x corresponding to a value of, not present in the table.

7. State Lagrange's inverse interpolation formula.

Answer :

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)}\,x_0 + \frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)}\,x_1 + \frac{(y-y_0)(y-y_1)\dots(y-y_n)}{(y_2-y_0)(y_2-y_1)\dots(y_2-y_n)}\,x_2 + \cdots \dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})}\,x_n$$

8. Define 'Divided Differences'.

Answer :

Let the function $y = f(x)$ take the values $f(x_0), f(x_1), \dots f(x_n)$ corresponding to the values $x_0, x_1, x_2, \dots \dots . x_n$ of the argument x where $x_1 - x_0,\ x_2 - x_1, \dots \dots x_n - x_{n-1}$ $need$ not necessarily be equal.

The first divided difference of $f(x)$ for the arguments x_0, x_1 is

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Similarly

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

9. Form the divided for the following data

x :	2	5	10
y :	5	29	109

Solution : The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
2	5		
		$\frac{29-5}{5-2} = 8$	
5	29		$\frac{16-8}{10-2} = 1$
		$\frac{109-29}{10-5} = 16$	
10	109		

9. Form the divided for the following data

x :	5	15	22
y :	7	3629	160

Solution : The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
5	7		
		$\frac{36-7}{15-5} = 2.9$	
15	36		$\frac{17.7-2.9}{22-5} = 2.114$
		$\frac{160-36}{22-15} = 17.7$	
22	160		

10. Give the Newton's divided difference formula.

Solution :

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)\,(x - x_1)f(x_0, x_1, x_2) + (x - x_0)\,(x - x_1)\,(x - x_2)f(x_0, x_1, x_2, x_3) + \cdots \ldots \ldots .. + (x - x_0)(x - x_1)\ldots(x - x_n)\,f(x_0, x_1, \ldots x_{n-1})$$

11. State any properties of divided differences.

Solution :

(1). The divide difference are symmetrical in all their arguments. That is the value of any difference is independent of the order of the arguments.

(2). The divided difference of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

12. State Newton's forward Difference formula to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$.

Answer :

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \cdots\right\} \quad and$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\ \Delta^4 y_0 - \cdots\right\}$$

13. Find the parabola of the form $y = a\,x^2 + bx + c$ passing through the points $(0,0), (1,1)$ & $(2,20)$.

Answer :

Let us known $f(x_0) = y_0$.

Here $y_0 = 0,\ \ \Delta\, y_0 = 1,\ \ \Delta\, y_1 = 19\,,\ \Delta^2\, y_0 = 18,\ p = x$

$$f(x) = 0 + x(1) + \frac{x(x-1)}{2}\,(18) \Rightarrow y = 9\,x^2 - 8x$$

14. Write the formula to compute $\frac{dy}{dx}$ at $x = x_0 + ph$ for the given data $(x_i, y_i), i = 0,1,2, \ldots n.$

Answer :

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\Delta y_0 - \frac{2p-1}{2}\ \Delta^2 y_0 + \frac{3p^2-6p+2}{6}\ \Delta^3 y_0 + \frac{2p^3-9p^2+11p-3}{12}\ \Delta^4 y_0 - \cdots\right\}$$

$where\ \ p = \frac{x-x_0}{h.}$

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15. Write the formula to compute $\frac{d^2y}{dx^2}$ at $x = x_0 + ph$ for the given data

$(x_i, y_i), i = 0,1,2, \dots n.$

Answer :

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\Delta^2 y_0 + (p-1)\,\Delta^2 y_0 + \frac{6p^2 - 18\,p + 11}{12}\ \Delta^4 y_0 + \cdots\right\}$$

$where\ \ p = \frac{x - x_0}{h.}$

16. State Newton's Backward interpolation formula to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_n$.

Answer :

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left\{\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \cdots\right\}\ \ and$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left\{\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\ \nabla^4 y_0 - \cdots\right\}$$

17. Write the formula to compute $\frac{dy}{dx}$ at $x = x_n + ph$ for the given data $(x_i, y_i), i = 0,1,2, \dots n.$

Answer :

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\nabla y_n - \frac{2p+1}{2}\ \nabla^2 y_n + \frac{3 + 6p + 2}{6}\ \nabla^3 y_n + \frac{2p^3 + 9p^2 + 11p + 3}{12}\ \nabla^4 y_0 - \cdots\right\}$$

$where\ \ p = \frac{x - x_n}{h.}$

18. Write the formula to compute $\frac{d^2y}{dx^2}$ at $x = x_n + ph$ for the given data

$(x_i, y_i), i = 0,1,2, \dots n.$

Answer :

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\nabla^2 y_n + (p+1)\,\nabla^3 y_n + \frac{6p^2 + 18\,p + 11}{12}\ \nabla^4 n + \cdots\right\}$$

$where\ \ p = \frac{x - x_n}{h.}$

19. Find $\frac{dy}{dx}$ at $x = 2$ from the following data.

$x:$	2	3	4
$y:$	26	58	112

Answer : $\Delta y_0 = 32, \Delta y_1 = 54\,, \Delta^2 y_0 = 22$

$$\frac{dy}{dx} = 32 - \frac{1}{2}\,(22) = 21.$$

20. Find $\frac{dy}{dx}$ at $x = 6$ from the following data.

$x:$	2	4	6
$y:$	3	11	27

Answer : $\nabla y_n = 16,\ \nabla y_{n-1} = 8,\ \nabla^2 y_n = 16 - 8 = 8$

$$\left(\frac{d^2y}{dx^2}\right)_{x=6} = \frac{1}{2}\left(16 + \frac{8}{2}\right) = 10.$$

21. A curve passing through the points $(1,0),(2,1)\ and\ (4,5)$. Find the slope of the curve at $x=3$.

Answer :

$$f(1,2)=1,\qquad f(2,4)=2,\qquad f(1,2,4)=\frac{1}{3}.$$

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)$$

$$f(x)=0+(x-1)(1)+(x-1)(\,x-2)\frac{1}{3}\ =\ x-1+\frac{1}{3}\,(x^2-3x+2)$$

$$f'^{(x)}=1+\frac{2x}{3}-1=\frac{2x}{3}$$

Slope at $x=3$ is $\frac{2(3)}{3}=2.$

22. State Trapezoidal rule with the error order.

Answer :

For the given data (x_i,y_i) where $x_i=x_0+ih,\ \ i=0,1,2\,\ldots.\,n$

$$\int_{x_0}^{x_n} y\,dx=\frac{h}{3}\,[(y_0+y_n)+2(y_1+y_3+\cdots+y_{n-1})]\qquad and$$

$$Error\ is\ of\ order\ h^2\,.$$

23. State Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule.

Answer :

If (x_i,y_i) $i=0,1,2,\ldots n$ where $x_i=x_0+ih,\ then$

Simpson's $\frac{1}{3}$ rule :

$$\int_{x_0}^{x_n} y\,dx=\frac{h}{3}\,[(y_0+y_n)+4(y_1+y_3+y_5+\cdots+)+2(y_2+y_4+y_6+\cdots)]$$

Simpson's $\frac{3}{8}$ rule :

$$\int_{x_0}^{x_n} y\,dx=\frac{3h}{8}\,[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+)+2(y_3+y_6+y_9\ldots)]$$

24. State the basic principle for deriving Simpson's $\frac{1}{3}$ rule.

Answer :

The curve passing through the consecutive points is replaced by a parabola.

25. State the order of error in Simpson's $\frac{1}{3}$ rule.

Answer :

Error in Simpson's $\frac{1}{3}$ rule is of order h^4.

26. Using Simpson's rule, find $\int_0^4 e^x\,dx$ given $e^0=1, e^1=2.72,\ e^2=\ 7.39,\ e^3=20.09$ &

$e^4=54.6.$

Answer :

$$\int_0^4 e^x\,dx = \frac{1}{3}\,[(1+54.6)+4(2.72+20.09)+2(7.39)] = 53.873.$$

27. A curve passes through $(2,8)$, $(3,27)$, $(4,64)$ & $(5,125)$. find the area of the curve between x- axis and the line $x = 2$ *and* $x = 5$, by Trapezoidal rule.

Answer :

$$\int_2^5 y\,dx = \frac{1}{2}\,[(8+125)+2(27+64)] = 157.5\ sq.units.$$

28. Find $\int_{-2}^{+2} x^4\,dx$ by Simpson's rule, taking h=1.

Answer :

$$\begin{array}{cccccc} x: & -2 & -1 & 0 & 1 & 2 \\ y: & 16 & 1 & 0 & 1 & 16 \end{array}$$

$$\therefore \int_{-2}^{+2} x^4\,dx = \frac{1}{3}\,[(16+16)+4(2)] = 13.3\ sq.units.$$

29. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal rule with $h = 0.5$.

Answer :

$$\int_0^1 \frac{dx}{1+x^2} = \frac{0.5}{2}\,[1.5+2(0.8)] = 0.775.$$

30. Use Simpson's $\frac{1}{3}$ rule with $h = 0.5$ to evaluate $\int_0^1 \frac{dx}{1+x}$.

Answer :

$$\int_0^1 \frac{dx}{1+x} = \frac{1}{6}\left[1+\frac{4}{1.5}+\frac{1}{2}\right] = 0.6944.$$

31. State the errors & order for Simpson's rule and Trapezoidal rules.

Solution :

Rule	**Degree of $y(x)$**	**No.of Intrevals**	**Error**	**Order**
Trapezoidal rule	One	Any	$\lvert E\rvert < \frac{(b-a)h^2}{12}M$	h^2
Simpson's $1/3$rule	Two	Even	$\lvert E\rvert < \frac{(b-a)h^4}{180}M$	h^4
Simpson's $3/8$ rule	Three	Multiple of 3	$\lvert E\rvert = \frac{3}{8}h^5$	-

PART - B

1. Find y' , y'' & y''' at $x = 1.05$ for the following data :

x :	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$y(x)$:	1.0000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

2. Compute $f'(0), f''(0),$ $f'(4)$ from the data.

x :	0	1	2	3	4
$f(x)$:	1	2.718	7.381	20.086	54.598

3. Consider the following table of data :

x :	0.2	0.4	0.6	0.8	1.0
$f(x)$:	0.9798652	0.9177710	0.8080348	0.6386093	0.3843735

   Find $f'(0.25)$ using Newton's Forward difference approximation, and $f'(0.95)$ using Newton's backward difference approximation.

4. Using Trapezoidal rule, Simpson's rule evaluate $\int_{-1}^{+1} \frac{dx}{1+x^2}$ taking 8 intervals.
5. Evaluate $\int_{0}^{+1} \frac{dx}{1+x^2}$ with $h = \frac{1}{6}$ by Trapezoidal rule & Simpson's rule.
6. Evaluate $\int_{0}^{5} \frac{dx}{4x+5}$ by Simpson's rule and trapezoidal rule, hence find the value of $\log_e 5$ $(n = 5)$.
7. Evaluate $\int_{0}^{6} \frac{dx}{1+x^2}$ by (1). Trapezoidal rule (2). Simpson's rule. Also check by actual integration.
8. By dividing the range into ten equal parts, evaluate $\int_{0}^{\pi} \sin x \; dx$ by Trapezoidal rule and Simpson's rule. Verify the answer by actual integration.
9. Evaluate $\int_{-1}^{+1} \frac{1}{1+x^2} dx$ by (1). Trapezoidal rule (2). Simpson's rule.
10. Evaluate $\int_{0}^{2} \frac{x^2+2x+1}{1+(x+1)^2} dx$ by (1). Trapezoidal rule (2). Simpson's rule.
11. Evaluate $\int_{-1}^{+1} \frac{x^2}{1+x^4} dx$ by using (1). Trapezoidal rule (2). Simpson's rule. Take $h = 0.2$
12. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$ from the following data :

x :	50	60	70	80	90
$y(x)$:	19.96	36.65	58.81	77.21	94.61

13. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x:	0	1	2	5
$f(x)$:	2	3	12	147

14. Using Lagrange's interpolation formula to calculate the profit in the year 2000 from the data.

$Year$:	1997	1999	2001	2002
$Profit$:	43	65	159	248

15. Find the third degree polynomial $f(x)$ satisfying the following data.

x:	1	3	5	7
$f(x)$:	24	120	336	720

16. Using Lagrange's interpolation formula find $y(2)$ from the following data.

x:	0	1	3	4	5
$f(x)$:	0	1	81	256	625

17. Using Lagrange's interpolation formula find $f(4)$ given that

$f(0) = 2,\ f(1) = 3,\ f(2) = 12,\ f(15) = 3587$

18. Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula.

x:	- 4	- 1	0	2	5
$f(x)$:	1245	33	5	9	1335

19. Using Newton's divided difference formula, find $u(3)$ given that

$u(1) = -26,\ u(2) = 12,\ u(4) = 256,\ u(6) = 844.$

20. Find $f(8)$ Newton's divided difference formula from the data :

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

21. Using Newton's divided difference formula, find $f(3)$ from the data

x:	0	1	2	4	5
$f(x)$:	1	14	15	5	6

22. Using Newton's divided difference formula, find the missing value from the table

x:	1	2	4	5	6
$f(x)$:	14	15	5	--	9