SEQUENCING MODELS AND NETWORK MODELS

A network is a symbolic representation of the essential characteristics of a project. PERT and CPM are the two widely applied techniques.

(a) Programme Evaluation and Review Technique (PERT)
It uses event orient network in which successive events are joined by arrows. It is preferred for projects that are non-repetitive and in which time for various activities cannot be precisely pre-determined. There is no significant past experience to guide; they are once-through projects. Launching a new product in the market by a company, research and development of new war weapon, launching of satellite, sending space craft to Mars are PERT projects. Three time estimates-the optimistic time estimate, pessimistic time estimate and the most likely time estimate are associated with each and every activity to take into account the uncertainty in their times.

(b) Critical Path Method (CPM)
It uses activity oriented network which consists of a number of well recognized jobs, tasks or activities. Each activity is represented by arrow and the activities are joined together by events. CPM is generally used for simple, repetitive types of projects for which the activity times and costs are certainly and precisely known. Projects like construction of a building, road, bridge, physical verification of store, yearly closing of accounts by a company can be handled by CPM. Thus it is deterministic rather than probabilistic model.

Activity
It is physically identifiable part of a project which requires time and resources for its execution. An activity is represented by an arrow, the tail of which represents the start and the head, the finish of the activity. The length, shape and direction of the arrow has no relation to the size of the activity.

Event
The beginning and end points of an activity are called events or nodes. Event is a point in time and does not consume any resources. It is represented by a circle. The head event, called the jth event, has always a number higher than the tail event, called the ith event i.e., j>i.

Path
An unbroken chain of activity arrows connecting the initial event to some other event is called a path.

Network

It is the graphical representation of logically and sequentially connected arrows and nodes representing activities and events of a project. Networks are also called arrow diagrams.

Network Construction

Firstly the project is spilt into activities. Start and finish events of the project are then decided. After deciding the precedence order, the activities are put in a logical sequence by using the graphical notations. While constructing the network, in order to ensure that the activities fall in a logical sequence, following questions are checked:

(i) What activities must be completed before a particular activity starts?
(ii) What activities follow this?
(iii) What activities must be performed concurrently with this?

Activities which must be completed before a particular activity starts are called the predecessor activities and those which must follow a particular activity are successor activities.

While drawing the network following points should be kept in mind:

1. Each activity is represented by one and only one arrow. But in some situations where an activity is further subdivided into segments, each segment will be represented by a separate arrow.
2. Time flows from left to right. Arrows pointing in opposite direction are to be avoided.
3. Arrows should be kept straight and not curved.
4. Angles between the arrows should be as large as possible.
5. Arrows should not cross each other.
6. Each activity must have a tail and a head event. No two or more activities may have the same tail and head events.
7. An event is not complete until all the activities flowing into it are completed.
8. No subsequent activity can begin until its tail event is completed.
9. In a network diagram there should be only one initial event and one end event.

Dummy

An activity which only determines the dependency of one activity on the other, but does not consume any time is called a dummy activity. Dummies are usually represented by dotted line arrows.

A dummy activity is introduced in the network for two reasons:

1. To maintain the precise logic of the precedence of activities. Such a dummy is called 'logical dummy'.
2. To comply with the rule that no two or more activities can have the same tail and head events. Such a dummy is called 'grammatical dummy'.

Looping (Cycling)

Sometimes due to faulty network sequence a looping arises. This situation can be avoided by checking the precedence relationship of the activities and by numbering them in a logical sequence.

Numbering the events (Fulkerson's rule)

The following steps involved in numbering the events were

1. The initial event which has all outgoing arrows with no incoming arrow is numbered '1'.
2. Delete all the arrows coming out from '1'. This will convert some more nodes into initial events. Number these events 2, 3, …
3. Delete all the going out from these numbered events to create more initial events. Assign the next numbers to these events.
4. Continue until the final or terminal node, which has all arrows coming in with no arrow going out, is numbered.

Merits and demerits of Activity On Node (AON) diagrams

The greatest merit of AON diagram is its simplicity. It is easier to draw, interpret, review and revise. Absence of dummies makes it more readily understood by non-technical users.

Difference between CPM and PERT

The projects which comprise of the variable type activities associated with probabilistic time estimates, employ PERT version of the networks and the projects comprising of deterministic type of activities are handled by CPM version of networks.

PERT is event-oriented while CPM is activity oriented.

Critical path analysis

The critical path of a network gives the shortest time in which whole project can be completed. It is the chain of activities with the longest time durations. These activities are called critical activities. They are critical in the sense that delay in any of them results in the delay of the completion of the project. There may be more than one critical path in a network and it is possible for the critical path to run through a dummy.

The critical path analysis consists of the following steps:

1. *Calculate the time schedule for each activity:* It involves the determination of the time by which an activity must begin and the time before which it must be completed. The time schedule data for each activity include the calculation of the earliest start, the earliest finish, the latest start, the latest finish times and the float.
2. *Calculate the time schedule for the completion of the entire project:* It involves the calculation of project completion time.
3. *Identify the critical activities and find the critical path:* Critical activities are the ones which must be started and completed on schedule or else the project may get delayed. The path containing these activities is the critical path and is the longest path in terms of duration.

Formula to calculate Earliest start time, Earliest finish time, Latest start time, Latest finish time, Total float, Free float, and Independent float

$$Earliest\ start\ time\ for\ intial\ node\ i\ in\ activity\ i \rightarrow j\ = 0$$

$$Earliest\ start\ time\ for\ node\ j\ in\ activity\ i \rightarrow j = \ ES$$
$$= Max\{Earliest\ start\ time\ of\ node\ i + \ t_{ij}\ \}$$

$$Latest\ finish\ time\ for\ final\ node\ j\ in\ activity\ i \rightarrow j$$
$$= Earliest\ start\ time\ of\ \ final\ node$$

$Latest\ finish\ time\ for\ \ node\ i\ in\ activity\ i \to j = LF$

$$= Min\{Latest\ finish\ time\ of\ node\ j - \ t_{ij}\}$$

$Earliest\ finish\ time = Earliest\ start\ time + t_{ij}$

$Latest\ start\ time = Latest\ finish\ time - t_{ij}$

$Total\ float = Latest\ finish\ time - Earliest\ finish\ time$

$$= Latest\ start\ time - Earliest$$

$Free\ float\ of\ activity\ i - j = Total\ float - (LF - ES) of\ event\ j$

$Independent\ float\ of\ activity\ i - j = Free\ float - (LF - ES) of\ event\ i$

Slack: It is the time by which occurrence of an event can be delayed. It is denoted by S and is the difference between the latest occurrence time and earliest occurrence time of the event.

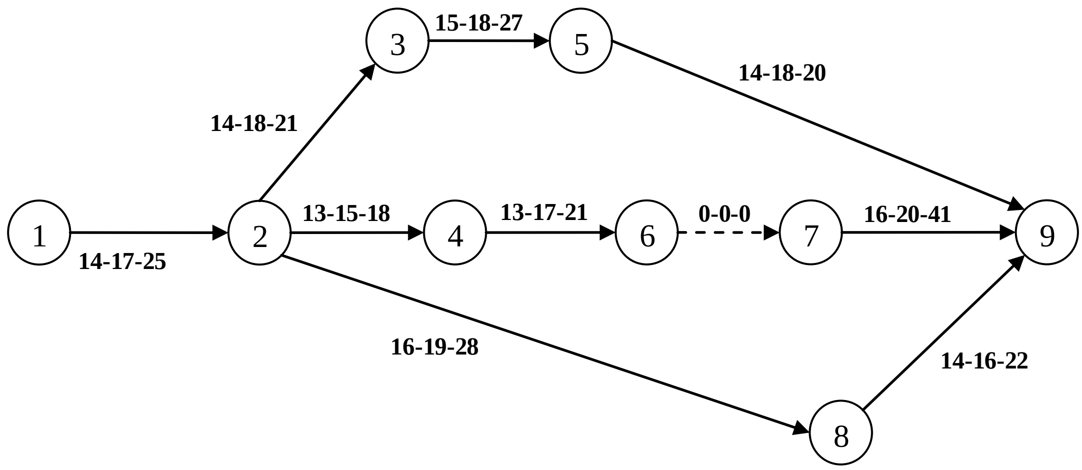
$S = LF - ES\ of\ the\ event.$

1. A civil engineering firm has to bid for the construction of a dam. The activities and their time estimates are given below:

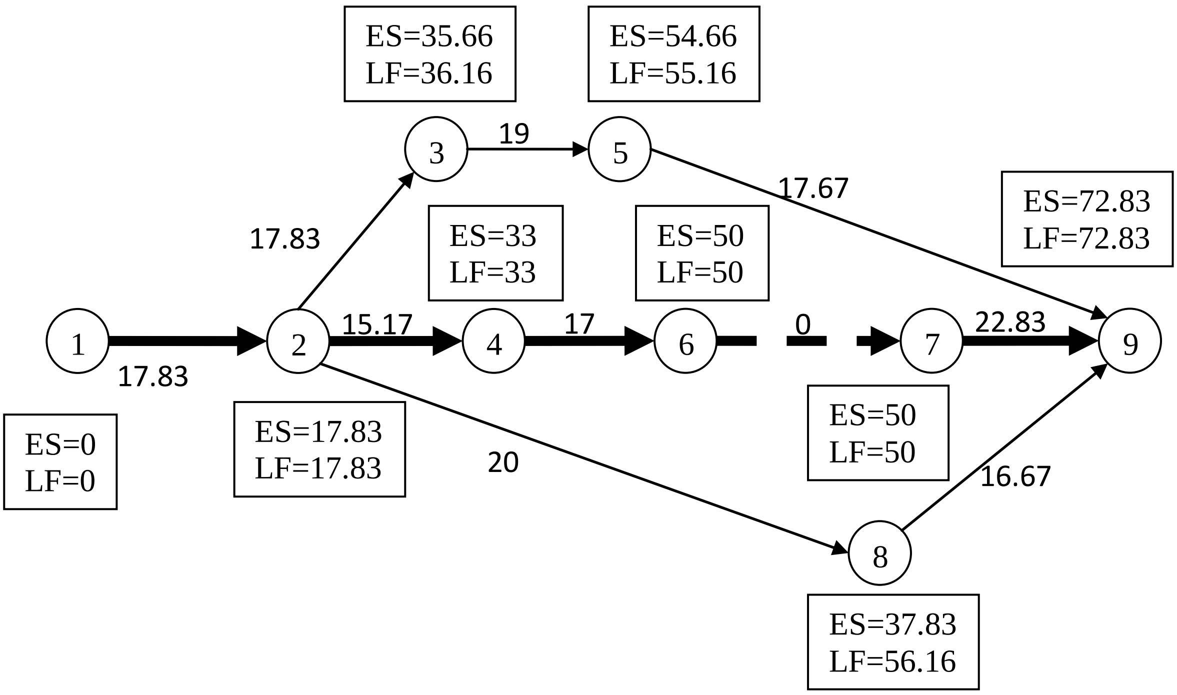
Activity :	(1-2)	(2-3)	(2-4)	(2-8)	(3-5)	(4-6)	(5-9)	(7-9)	(8-9)
Optimistic time :	14	14	13	16	15	13	14	16	14
Most likely time :	17	18	15	19	18	17	18	20	16
Pessimistic time :	25	21	18	28	27	21	20	41	22

The policy of the firm with respect to submitting bids the minimum amount that will provide a 95% of probability of at best breaking-even. The fixed costs for the project are eight lakhs and the variable costs are 9,000 every day spent on working on the project. The duration is in days and the costs are in rupees. What amount should the firm bid under this policy? (Table value P(Z=1.65)=0.95)

Solution:



Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1-2	14	17	25	17.83	**3.36**
2-3	14	18	21	17.83	1.36
2-4	13	15	18	15.17	**0.69**
2-8	16	19	28	20	4
3-5	15	18	27	19	4
4-6	13	17	21	17	**1.78**
5-9	14	18	20	17.67	1
6-7	0	0	0	0	**0**
7-9	16	20	41	22.83	**17.36**
8-9	14	16	22	16.67	1.78



Critical path is 1-2-4-6-7-9

Variance of critical path = 3.36+0.69+1.78+0+17.36=23.19

Standard deviation of critical path $\sigma = \sqrt{23.19} = 4.82$

Given that $P(Z = z) = 0.95$

$$z = \frac{X - \mu}{\sigma}$$

$$1.645 = \frac{X - 72.83}{4.82}$$

$$X = (1.645)(4.82) + 72.83$$

$$X = 80.76 \sim 81\ days$$

The fixed cost of the project is Rs. 8 lakhs and the variable cost is Rs. 9,000 per day.

$$The\ amount\ to\ bid\ = Rs.(800000 + 9000 \times 81) = Rs.15{,}29{,}000$$

2. The activity data for a project is given below:

Activity :	(1-2)	(1-3)	(1-4)	(2-5)	(2-6)	(3-6)	(4-7)	(5-7)	(6-7)
Optimistic time :	5	18	26	16	15	6	7	7	3
Pesimistic time :	10	22	40	20	25	12	12	9	5
Most likely time :	8	20	33	18	20	9	10	8	4

Determine the following:

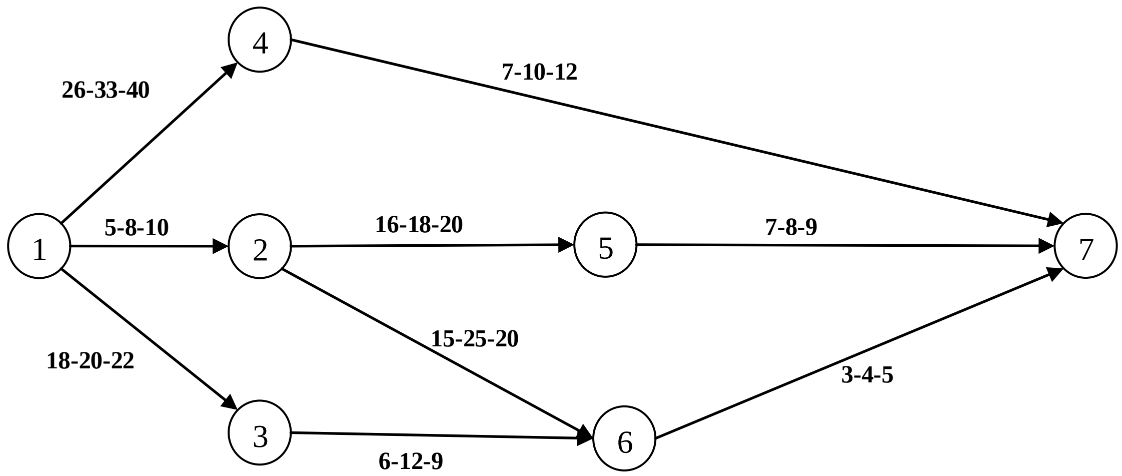
a) expected task times and variances.

b) the earliest and latest expected times to reach each node.

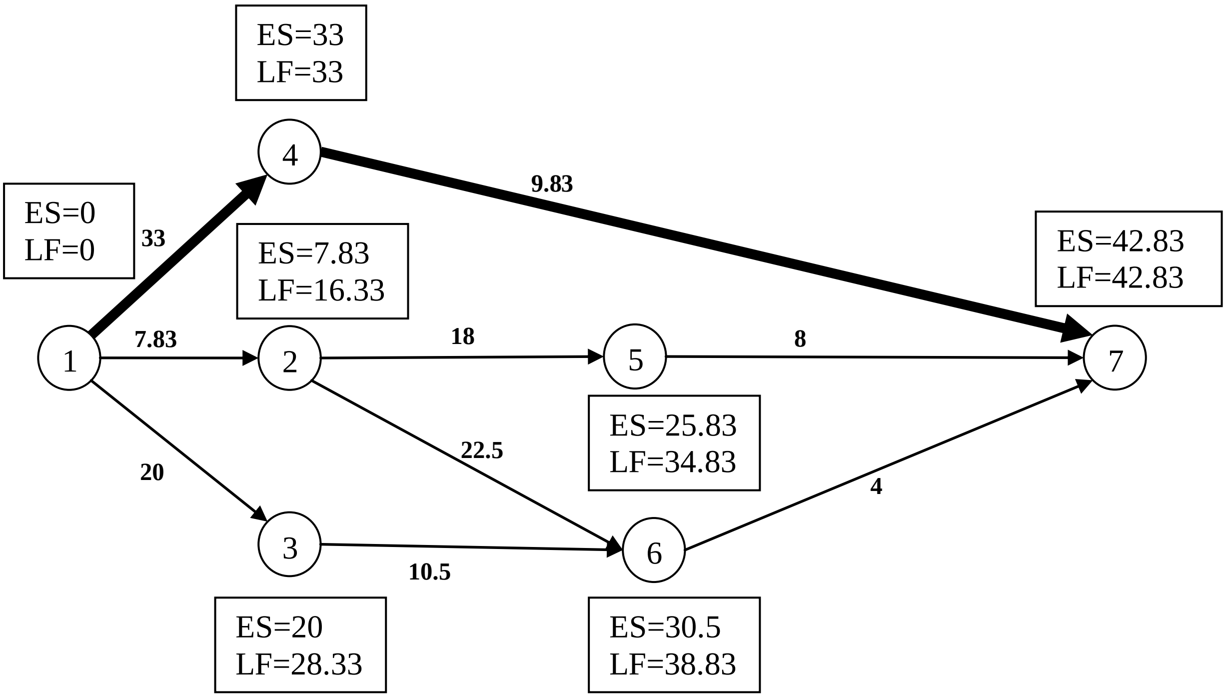
c) the critical path

d) the various paths and its project duration, and

e) the probability of node occurring at the proposed completion date if the original contract time of completing the project in 41.5 weeks. (Table value P(Z=z)=0.30)



Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1-2	5	8	10	7.83	0.69
1-3	18	20	22	20	0.44
1-4	26	33	40	33	**5.44**
2-5	16	18	20	18	0.44
2-6	15	25	20	22.5	0.69
3-6	6	12	9	10.5	0.25
4-7	7	10	12	9.83	**0.69**
5-7	7	8	9	8	0.11
6-7	3	4	5	4	0.11



Activity	t_e	Start time		Finish time		Total float
		Earliest	Latest	Earliest	Latest	
1-2	7.83	0	8.5	7.83	16.33	8.5
1-3	20	0	8.33	20	28.33	8.33
1-4	33	0	0	33	33	0
2-5	18	7.83	16.83	25.83	34.83	9
2-6	22.5	7.83	16.33	30.33	38.83	8.5
3-6	10.5	20	27.83	30.5	38.83	8.33
4-7	9.83	33	33	42.83	42.83	0
5-7	8	25.83	34.83	33.83	42.83	9
6-7	4	30.5	38.83	34.5	42.83	8.33

The critical path is 1-4-7
The variance of critical path is 5.44+0.69= 6.13
The standard deviation of the critical path is $\sqrt{6.13} = 2.48$

The various path and its project duration are as follows

1) $1-4-7 \Rightarrow \ 42.83$
2) $1-2-5-7 \Rightarrow \ 33.83$
3) $1-2-6-7 \Rightarrow \ 34.33$
4) $1-3-6-7 \Rightarrow \ 34.5$

The probability of node occurring at the proposed completion date if the original contract time of completing the project in 41.5 weeks.

$$Z = \frac{X - \mu}{\sigma} = \frac{41.5 - 42.83}{2.48} = -0.5363$$

$$P(Z = -0.5363) = 1 - 0.70 = 0.30$$

The probability of completing the project in 41.5 weeks is 30%.